Scottish Attainment Challenge
Cognitively Guided Instruction Project: 2016 – 2018

A report on supporting teachers' understanding of children's mathematical thinking

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Contents

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We would like to dedicate this work to the memory of Prof. Tom Carpenter. Without his pioneering research and inspirational contribution to the field none of this would have ever been possible.

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Photographs

The picture on the front cover of the report was taken by Cheryl Lubinski and shows a Primary 1 pupil sharing his solution strategy of a multiplication problem with peers.

All appropriate consents were obtained in relation to the taking and reproduction of photographs.

Executive Summary

> **"It is not just about the children's attainment, it is about upskilling the staff and those for me have been the two biggest impacts that I have seen in my school."** *Headteacher A, 2018*

What is Cognitively Guided Instruction?

Cognitively Guided Instruction (CGI) is a research-based approach to teaching mathematics that recognises children's intuitive knowledge of mathematics and their ability to make sense and to solve mathematical problems. Carpenter et al. (1999) argue that 'young children have quite different conceptions of addition, subtraction, multiplication and division than adults do' and set out how children's mathematical thinking develops and can be used to inform teaching practice.

Why this matters

Schools are currently under considerable pressure to raise attainment in numeracy. Our study suggests that a rise in attainment can come about by focussing on teachers' knowledge and pedagogy and specifically, teachers' pedagogical content knowledge, rather than by focussing on attainment itself.

Focussing on children's conceptual understanding does not need to be at the expense of computational fluency but teaching children in ways that ensure that they understand the maths they are learning can accomplish both. Many adults will struggle to recall how to calculate 2½ ÷ ¼, few will struggle to work out how many people can get quarter of a pizza if there are two and half pizzas. Teaching children in ways that allow them to connect their intuitive mathematical understanding to the formal mathematics of school is the task of the accomplished teacher. Our study identifies a model of professional learning to support teachers to develop their capacity in this role.

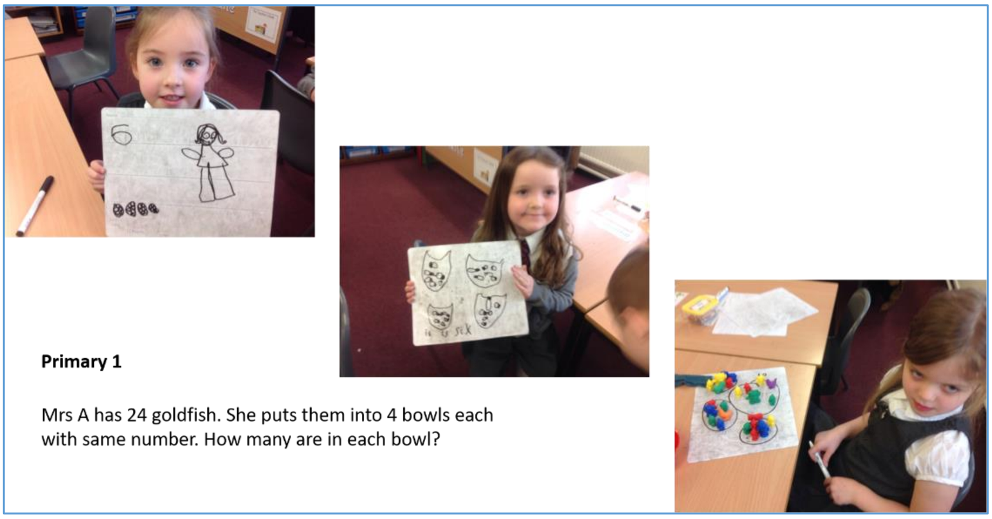

Figure 1: Extract from observation notes, Primary 1 strategies for solving a division problem

Developing a Model of Professional Learning

Teacher professional learning needs to

- be situated, meaningful, supported
- focus on children's thinking
- be inclusive
- focus on pedagogy
- be about knowledge
- be respectful of teachers' prior knowledge and ways of learning
- be mathematically informed
- be part of a professional learning community
- be long term

Figure 2: Organising principles for teacher professional learning

This project delivered a range of professional learning interventions for 75 teachers and 790 pupils across 3 primary schools in different local authority areas. We sought to inform teachers about the development of children's mathematical thinking and support changes in their classroom practice that would lead to improved attainment in children's numeracy.

Our intention was to develop an effective whole school model of professional learning that would inject new knowledge, stimulate new teacher moves and, over time, develop a professional learning community that could continue to support CGI in each of the three schools.

Working closely with CGI coordinators in each school, an introduction to CGI was delivered to all teachers through either accredited training or in-school seminars. It was followed up with interventions in the classroom, professional dialogue and CGI Lesson Study. The CGI coordinators became the lynch pin of CGI activity in their school, organising meetings, classroom access, teacher cover and on the spot advice and support for teachers in their school. Academics from both Scotland and the US who had studied with members of the original University of Wisconsin-Madison team and were experienced in primary school teaching, worked with teachers in their classrooms, focussing on children's learning. As the project progressed Lesson Study became the dominant model for supporting teachers' learning. It provided a collaborative and reflective framework to which teachers were highly responsive.

Impact on teachers

Over a period of time we witnessed a transformation of practice. Fundamentally this was evident in a shift away from what has been described as a transmission approach, towards a pedagogical orientation that supported children making connections in their mathematical understanding. Teachers reflected on their role and how instruction might be designed to support sense-making for pupils in tandem with computational fluency. Teaching became informed by children's thinking. Teachers were more aware of what to look for and they had a greater understanding of what they were seeing the children doing. They were beginning to use this to inform their teaching. Teachers

"The teachers' knowledge of mathematical thinking has developed so greatly and that has had a great impact on our children." *Headteacher B, 2018* recognised this was not a formulaic approach and committed considerable time and energy. The project findings are consistent with previous studies that suggest that increasing teachers' knowledge of children's mathematical thinking can improve pupil attainment, typically taking 2-3 years to build knowledge and skills in individual teachers to become confident practitioners and for longer periods, 5-7 years, for those seeking to lead learning within their school.

Impact on pupil attainment

There was a statistically significant rise in some areas of pupil attainment and an improvement in almost all other areas across all three schools evidenced in the pre- and post- intervention assessments. This was mirrored in school-based assessments and teacher judgements. Pupils became more engaged in mathematics, showing increased confidence, independence and flexible thinking. They were able to apply what they already knew to help them tackle problems of increasing complexity. Pupils in upper primary classes rely most on teachers supporting them to make links between their own strategies and what are considered the standard algorithms seen as required for secondary education.

> **"It's a more holistic approach to assessing and it's an ongoing process rather than a one-day snapshot of their achievements. What the data is telling us is that there has been a massive increase in the attainment of numeracy across the whole school."** *Headteacher C, 2018*

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Chapter 1: Introduction

This report presents the findings of a project which ran from April 2016 until June 2018 and involved supporting teachers in learning about children's mathematical thinking and using that knowledge to inform their teaching through a professional development programme of Cognitively Guided Instruction (CGI). The report aims to bring together the evidence on pupil attainment with reflections on the professional development interventions deployed as the project progressed, starting first at Hareleeshill Primary School from April 2016, followed by St Stephen's and Bonhill Primary Schools from June 2017.

Background

The project took place in three schools in different Local Authorities: Hareleeshill Primary, South Lanarkshire; St Stephen's Primary, North Lanarkshire and Bonhill Primary, West Dunbartonshire. In each case the initial request to develop Cognitively Guided Instruction (CGI) within the schools came directly from the headteachers. At the end of 2014 the headteacher of Hareleeshill Primary, South Lanarkshire, undertook an introductory course in CGI led by Dr Lio Moscardini at the University of Strathclyde. On the basis of what she learned, the headteacher believed that all teachers should have this understanding and she was keen to develop CGI across Hareleeshill Primary, consequently in 2015 seven teachers from Hareleeshill Primary School undertook the course. In September 2016 a group of teachers at St Stephen's Primary, including the headteacher and depute headteachers, and a teacher from Bonhill Primary undertook the course. As a result of what was witnessed happening in the classrooms, the headteachers in both schools sought to develop CGI more fully across their whole school. Scottish Attainment Challenge and Pupil Equity Fund funding enabled us to draw together and develop work which had already commenced in Hareleeshill to include all the three schools in a focussed and strategic way. This meant that by June 2017 we were working with a cohort of 75 teachers and 790 pupils. The project included pre and post-intervention interviews with staff as well as pre and post-intervention assessments of pupils' learning.

What is Cognitively Guided Instruction?

Cognitively Guided Instruction (Carpenter et al., 1999) is a professional development programme developed at the University of Wisconsin-Madison over a period of nearly twenty years through cyclical research and the application of findings to practice. Cognitively Guided Instruction recognises children's intuitive knowledge of mathematics and their ability to make sense and to solve mathematical problems. Carpenter et al. (1999) argue that 'young children have quite different conceptions of addition, subtraction, multiplication and division than adults do' and set out how children's mathematical thinking develops and can be used to inform teaching practice. Professional Development in CGI builds on this by supporting teachers to provide an environment, opportunity and encouragement for children to construct problem-solving strategies that are first of all meaningful to them and in which teachers then support pupils to find the most effective and efficient strategies. CGI focuses on the following elements:

- the development of children's mathematical thinking
- instruction that influences this development
- teachers' knowledge and beliefs that influence their instructional practices

- the way that teachers' knowledge, beliefs and practices are influenced by their understanding of children's mathematical thinking

(Carpenter, Fennema, Franke, Levi & Empson, 2000, p.1)

In CGI it is recognised that increasing teachers' knowledge of students' thinking helps them to design better instructional tasks and to support individual student learning more effectively. In practice, CGI includes the use of arithmetical word problems, although it is not restricted exclusively to word problems. Teachers are provided with two related research-based frameworks: word problem types and children's solution strategies. As pupils engage with particular problems teachers learn to interpret their solution strategies and then use this analysis to inform their teaching. In this way teaching follows constructivist principles and is based on building on the sense that children are making of problems; teachers focus on what students know and understand and through a guided approach help them to build on that understanding.

In CGI word problems are used as a basis for introducing and developing mathematical concepts rather than as a means of determining whether children can apply existing abstract number knowledge to a word problem situation. An essential theme that emerged from earlier studies (Carpenter et al., 2000) is that children come to school with intuitive knowledge of mathematics that they can use to solve word problems without formal or explicit instruction on specific number facts and procedures. They do this by following the language of the problems initially by direct modelling, that is by following the language of the problem and modelling the action of the problem. For example in a joining problem such as - 'There are 4 children on the bus. The bus stops at a bus stop and 3 more children get on the bus. How many children are on the bus now?'- a child would physically represent the four children with cubes, drawings or fingers and would then physically represent the three children getting on the bus. The child would then join both sets and count from one to find the total. With time this direct modelling is replaced by more efficient counting strategies which in turn give way to the use of number facts. In this way the learning of number facts is not a rote skill but rather is built on an understanding of the relationships between numbers developed through these modelling and counting strategies (Carpenter et al., 1999, p.4). Through attending to the structure of the problems in this way, children engage with important mathematical ideas and develop basic concepts of addition, subtraction, multiplication and division. They can then build on this intuitive mathematical knowledge and construct concepts of place value and multidigit computational procedures. Carpenter et al. (ibid) outline the following strategies that children may use to solve problems:

- Direct Modelling: using manipulatives, fingers or drawing to follow the story of the problem exactly to model out a solution. In direct modelling the numerosity of sets within a problem are represented
- Counting: problems are solved by employing a range of counting strategies manipulatives, fingers or tally marks may be used to keep track of counting
- Derived facts: using known number facts to solve problems involving unknown facts
- Recall: recalling known number facts

(from Moscardini, 2010, p.131)

An important aspect of CGI is that it is not a prescriptive pedagogy or an acquirable teaching technique. It is a principled approach to teaching mathematics. It represents a situationally-based form of professional development, a feature of which is the latitude that teachers have to develop their own learning and understanding of children's mathematical thinking. This openness is particularly relevant given current curricular developments in Scotland. It differs from the wide-spread tradition of procedural mastery of algorithmic skill that Maclellan (2014) describes as worldwide. In contrast, CGI requires of teachers both broader pedagogical knowledge about children's mathematical development and deeper, conceptual subject knowledge as well as direct experience in recognising and interpreting children's strategies and potential development trajectories. Teachers learn to focus on children's understanding and this provides a context for teachers to develop their own pedagogical knowledge. Thus teacher learning becomes a dynamic process situated within classroom interactions and interpretations. Professional development to support the introduction of CGI has therefore to challenge beliefs about the mathematical understanding children bring to the classroom. It needs to encompass both pedagogical and subject matter knowledge and how this comes to be integrated in pedagogical content knowledge and it needs to be situated in the classroom (Moscardini, 2014).

The development of CGI in Scotland

To date there has been little development and research of CGI in classrooms in the UK, research studies have been almost exclusively American. There is a growing body of US published research which evidences the positive impact that CGI has in developing teachers' knowledge, specifically in terms of pedagogy and knowledge of students' mathematical thinking and increased pupil outcomes. However, in the UK there are only two peer-reviewed research studies. Both are comparatively small-scale and both have positive results; one involved the development of CGI with a group of 21 mainstream primary teachers (Moscardini, 2014) and the other involved CGI development in schools for children with learning difficulties (Moscardini, 2015; 2010; 2009).

Dr Moscardini began working with CGI in his own classroom practice in 2003. In 2005 he undertook the Advanced Institute in CGI at the University of Wisconsin-Madison with members of the original research team including Prof Tom Carpenter, Dr Linda Levi, Sue Gehn and Annie Keith; he is the only person in the UK to have undertaken this study. His study of CGI has been ongoing with frequent and regular visits to the US. He has also been invited regularly to the US to present on the work he is involved in in Scotland. In 2012 he developed an introductory course on CGI for Scottish teachers. He has led professional development in CGI with teachers in various Scottish local authorities as well as in Switzerland and Italy.

One of the challenges in developing CGI is that it is recognised that it takes two to three years for teachers to develop a sound understanding of CGI and about seven years with ongoing study in order to lead professional development in CGI (Jacobs, Lamb & Phillip, 2010; Franke et al. 2001; Fennema et al., 1996). One of the problems that is encountered is a tendency for a quick-fix approach to developing practice based on a cascade model often led by teachers who have not had the time or support to develop their own practice and to consolidate their own knowledge and understanding. For example, in the Education Scotland School Improvement Partnership Project run by the Robert Owen Centre, University of Glasgow, a few years ago, the final report notes the importance of professional development in CGI being led by knowledgeable and experienced practitioners. The report states that 'some of the work distributed by the schools [in the SIPP project] evidenced teachers' misunderstandings about important aspects of children's mathematical thinking which would affect children's learning. This was not unusual given the limited experience of CGI of the teachers leading discussions.' This problem has been identified as systemic, with a US study showing evidence of weak knowledge and pedagogical beliefs of leaders, at school and district level, which could hamper effective development (see Nelson, Stimpson & Jordan, 2007).

In our project, we placed significant emphasis on the importance of developing a sustainable model which would maintain the integrity of the principles of CGI. This would require supporting teachers to build deep knowledge which might transform practice. We considered it essential that teachers should be supported in their learning by highly experienced and knowledgeable practitioners of CGI and that this learning should be situated in classrooms. Rather than opting for a 'mile wide and an inch deep' approach, we worked in depth with a small number of schools with the aim of building in deep and lasting knowledge over time and which could be sustained in practice. This model supports the development of future CGI leaders within and potentially beyond those schools and allows for a more authentic and sustainable scaling up. It reflects the effective professional development model practised and observed by us in some US districts.

Why introduce CGI to primary school numeracy education in Scotland?

One feature of numeracy education illustrated by Curriculum for Excellence (CfE) Results (Scottish Government 2017) and earlier surveys (e.g. PISA and TIMSS), has been a notable decline in the percentage of pupils achieving at each level as they progress through primary school. One explanation of this that CGI might address could be that children are finding themselves less able to make sense of mathematics as they progress through school and find themselves increasingly challenged by procedures they do not fully understand. The apparent improvements perceived in early secondary results (Scottish Government, 2017a) might equally suggest that teacher education and subject knowledge may also play a part.

The disparity in performance as pupils progress through the various levels is even more marked for the most disadvantaged pupils and in those primary schools serving the most disadvantaged communities. This project to introduce CGI to primary school numeracy education has therefore been financially supported by the Scottish Attainment Challenge and Pupil Equity Funding.

Chapter 2: Aims & Objectives of the CGI Project

The overarching aim of the project has been to support teachers to improve achievements in numeracy, particularly for the most disadvantaged pupils in primary education.

The project has both professional development and research aims. In terms of professional development, the project aims to improve school performance by working with UK and US based experts in CGI to build learning communities in a small number of schools where teachers can develop and share their CGI experience. Its research aims are to identify the most effective means of generating transformative learning opportunities in CGI for teachers and pupils.

Professional Development Aims

Professional development in this project builds closely on work in the US (Carpenter et al., 2000; Carpenter et al. 1999; Jacobs, Lamb & Philipp, 2010; Fennema et al., 1993) that highlights key learning points for teachers:

- knowledge about children's intuitive understanding of mathematics and how children's mathematical thinking develops
- word problems as a way of presenting mathematical problems to pupils and how the different types of word problems link to children's mathematical thinking
- a range of solution strategies commonly used by children and how different materials can support children to find their own solution strategies
- meaningful dialogue about solution strategies
- developing efficient solution strategies that prepare pupils for problems at the next level
- attending to and interpreting children's solution strategies to inform instruction

The learning points identified above suggest that CGI requires a radical shift away from traditional transmission educational models to a constructivist and transformative model that requires an unusually high degree of autonomous thinking and planning by teachers. To achieve this, teachers have to be inspired to learn and change – to have an inquiry-based attitude (Meijer et al., 2017; Schon, 1991).

WORD PROBLEM TASK: There are 6 children wearing shoes. How many shoes are there? — Primary 1-2

This pupil uses the ten rod to represent a child and the units rod to represent the 12 shoes.

This girl draws 6 children and puts 2 shoes on each to get 12

Figure 3: Extract from Research Team Primary 1 Observation Notes

Teachers need to feel safe and be sufficiently confident to experiment, to trial new approaches in the classroom and develop professional, reflective conversations with expert facilitators or colleagues (Coe et al. 2014; Mezirow & Taylor, 2009; Schuck, Aubusson & Buchanan, 2008). They need to be supported by management to work together, allowed time to practice (Argyris, 2004) and develop new teaching repertoires that may depart from more traditional planners.

From the outset of the project we adopted the same constructivist philosophy which underpins CGI to supporting teachers in their learning, anticipating experimentation and practice to make sense of the new knowledge brought to teachers (Lambert et al., 2002). In any whole school approach among the teachers there will be a range of attitudes and levels of interest/motivation in acquiring new knowledge and skills around CGI. Nonetheless, we believe that the opportunities afforded by significant investment in professional development, the attention and input of senior academics and the engagement of colleagues in skills development provide the inspiration, knowledge, situated and social learning opportunities necessary to transform their teaching practice.

The research aims to determine some organising principles for developing CGI practice in a whole school setting, identifying relevant context, dilemmas and debates that inhibit or support transformative learning among teachers.

Research Aims

In line with aim of the Scottish Attainment Challenge the study explored if developing teachers' understanding of children's mathematical thinking through professional development in CGI led to an increase in attainment for children in a primary school in areas of low socio-economic status. The following related themes were also explored over the course of the project:

- children's conceptual understanding
- teachers' learning
- issues inclusion and equity
- fidelity of professional development

Research Questions

The specific research questions of the project were:

RQ 1 Following professional development in CGI, is there evidence of an increase in attainment for children across the primary school?

RQ 2 Is there evidence of growth in teachers' knowledge of children mathematical thinking following professional development in CGI?

RQ 3 What factors contributed to any growth (if any) in teachers' knowledge of children's mathematical thinking?

RQ 4 Is there evidence of growth in children's conceptual understanding in numeracy?

RQ 5 What is the nature of any relationship between children's engagement in mathematical activity and teachers' pedagogical practices?

RQ 6 To what extent, if at all, are the affordances of CGI responsive to issues relating to inclusion and equity?

Chapter 3: Theory of Change

Modelling the project theory of change

The project sought to impact on pupil attainment through a range of professional development interventions for teachers that would introduce new knowledge, impact on their understanding of children's mathematical development and the moves teachers use to elicit learning in their classrooms. Using their new understanding of children's mathematical thinking, teachers should be able to design instruction that is more effective and improve pupil attainment. The figure below summarises the project's theory of change.

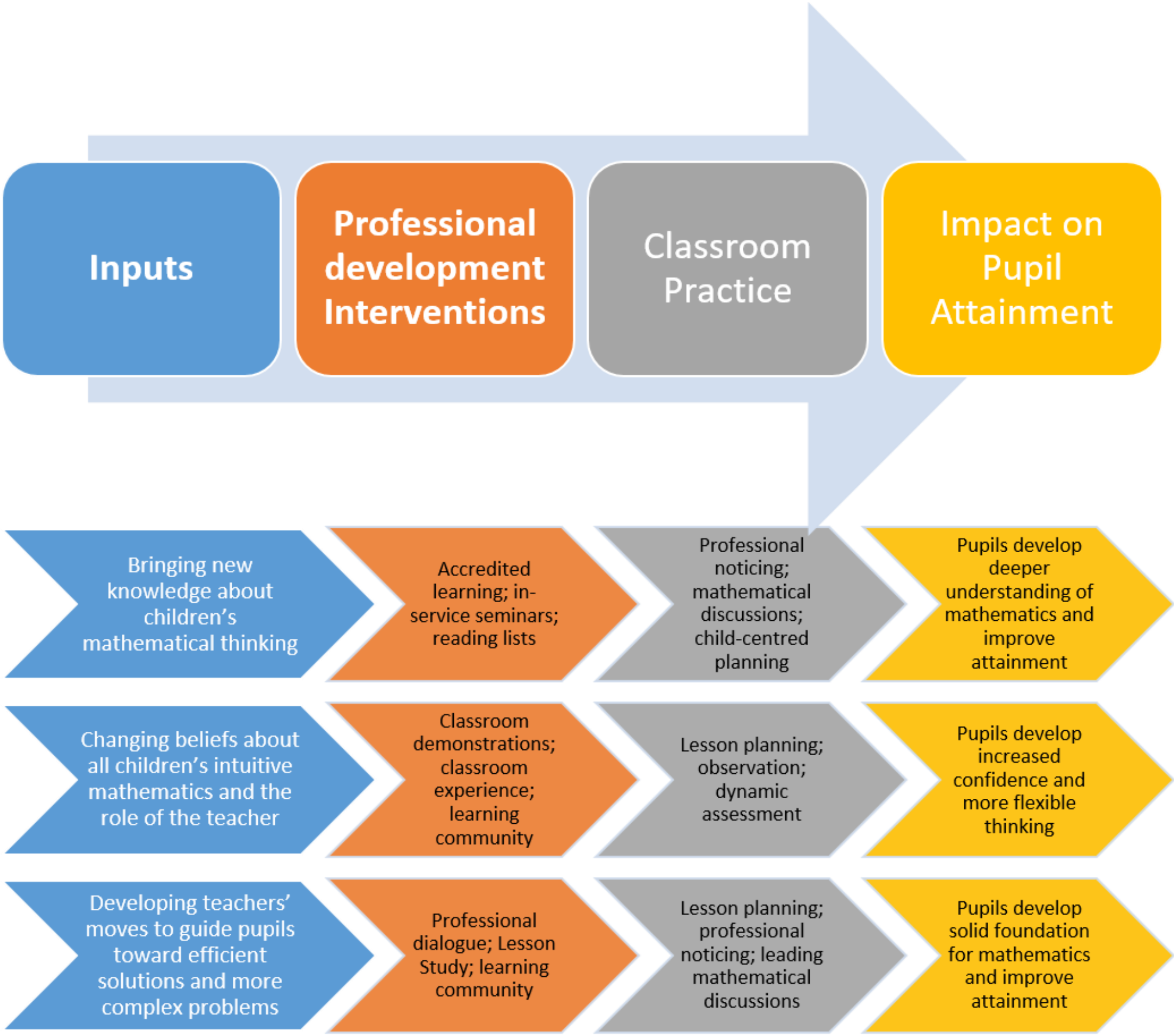


Figure 4: CGI Professional Learning Theory of Change

Teachers’ knowledge & practice

Mathematical knowledge for teaching

A not infrequent recommendation for the improvement of children’s mathematical understanding is that primary teachers’ own mathematical skills should be further developed. This is a controversial but not unfounded proposition not least with some primary teachers who feel that their professional qualification along with the entry requirements of initial teacher education evidences their suitability in this respect. Nevertheless, some primary teachers do express lack of confidence in their own mathematics and there is evidence that the mathematical understanding of some teachers is not all that it should be (see for example, Lubinski, Otto & Moscardini, 2017). However, the suggestion that student outcomes can be raised through better teaching simply as a direct consequence of having teachers who are better at maths themselves is naïve. Consideration needs to be given to the particular kinds of mathematical knowledge required for teaching and how this knowledge relates to pedagogy. In our work we have drawn heavily on the work of Deborah Ball, Lee Shulman and others in this respect.

In simplified terms there are three key areas in which teachers require to be knowledgeable. They need sound mathematical knowledge and understanding; knowledge of teaching, mathematical content and the curriculum; and knowledge of students in both a general and specific sense. They need knowledge of children’s development in numeracy and their mathematical thinking both in relation to all children as well as at the level of an individual child. We argue that in initial teacher education the focus has traditionally been on developing the first two of these areas: teachers’ mathematical subject knowledge and teachers’ knowledge of pedagogy and the curriculum. In other words, if teachers have proficiency in both maths and the general pedagogical skills of teaching then that in itself is sufficient. This is a limited and restricted view of teaching and omits the crucial place of knowledge of children’s mathematical thinking and how this might come to inform instruction. Fundamentally, CGI is about using knowledge of children’s thinking to inform teaching.

Pedagogical content knowledge

A more elaborate mapping of mathematical knowledge for teaching is at Figure 5. This domain map, developed by Hill, Ball & Schilling (2008) was an important element of the conceptual framework of the project and was particularly influential in supporting the workshop activities and Lesson Study development during the project. A detailed account of the various elements is set out in the original paper (Hill, Ball & Schilling, 2008, pp.377-378). Briefly, the left-hand side of the figure describes the kind of mathematical knowledge common across professions as well as that which is specialized to teaching, however it does not include knowledge of students or teaching. This knowledge of students’ thinking and how it intertwines with the pedagogical knowledge required for teaching is central to Shulman’s concept of pedagogical content knowledge (Shulman, 1986).

> *‘Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons’.*
> *(Shulman, 1986, p.9)*

Pedagogical content knowledge is recognised as a complex construct and in relation to mathematics teaching the mathematical component is a crucial one (Depaepe, Verschaffel & Kelchtermans, 2013;

Ball et al., 2008). It is important to bear this in mind particularly at a time when, arguably, developments in practice may support the engagement of children in particular activities but with insufficient attention to the actual mathematical engagement. In an important large-scale study which sought to understand effective teachers in numeracy led by Prof. Mike Askew and Prof. Margaret Brown, the researchers set out a model which acknowledged the different aspects of teachers' pedagogical content knowledge (PCK) as posited by Shulman (1986). Their framework was characterized by teachers' subject knowledge, their knowledge of teaching approaches and their knowledge of learners, and how these come together to inform instruction (Askew et al., 1997, p.18). In our project we focussed on developing teachers' pedagogical content knowledge.

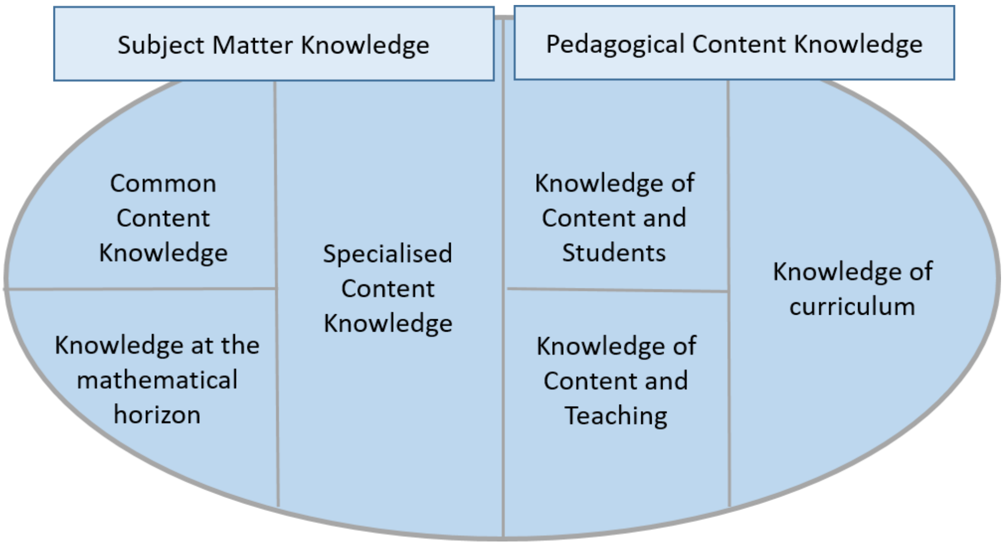


Figure 5: Domain map for mathematical knowledge for teaching (Hill, Ball & Schilling, 2008)

Inclusive practice

While an important aim of the project was to support the development of inclusive practice and this was clearly stated in our research questions, it did not feature as an explicit element of the professional development activity with teachers. Furthermore, we eschewed remedial approaches to intervention based on a medical model which fails to take into account pedagogy and the learning environment as potential sources of some children's difficulties. Our implicit proposition was that by developing teachers' pedagogical content knowledge and supporting the application of this in their classroom practice they would be better equipped to support all learners. However there is a concern that this knowledge base is not seen as relevant to the learning of all pupils with a view that some additional or different pedagogical knowledge is required for some learners as opposed to recognising what is common and available to all learners and how this might be extended (Ylonen & Norwich, 2012; Florian, 2009).

A traditional response to support children who struggle in their learning follows a medical model in which the problem is viewed as a deficit within the child to be remediated. We argue that these remedial approaches to educational support are deep-rooted in the educational system and are being maintained and in fact promoted in some initial teacher education institutions through clinical models of practice which fail to take into account pedagogy and the learning environment as factors which give rise to a need for support (see Scottish Government, 2017b). Such reductionist approaches are fundamentally rooted in behaviourist, lock-step approaches to teaching and assessment that historically have been a feature of special education (Thomas & Loxley, 2007; Dyson, 2001; Goddard, 1997). Teachers and leaders within schools might consider how they conceptualize learning difficulties not solely in terms of the needs of the individual but also from a pedagogical perspective (Tomlinson, 2017; Swann et al., 2012; Dyson, 2001). It has been argued that the individualization which traditionally lies at the heart of many interventions designed to respond to children's needs is problematic because it encourages a focus on the individual rather than on the curriculum, thus failing to recognize issues of instruction as potentially problematic (Ainscow et al., 2006). A pedagogy in which the 'transformability' of every learner is recognised supports the learning capacity of every individual and the development of an inclusive culture (Swann et al., 2012; Hart et al., 2004). Such practice also requires domain specific knowledge without which teachers may be ill-equipped to support all learners (Ball, Thames & Phelps, 2008; Ma, 1999; Hiebert et al. 1997).

Our project assumed an interactionist position which recognizes the importance of learning and development from the point of view of the teacher as well as that of the child. In practice this means that concerns about meeting a child's needs through a process of assessment which is perhaps driven by determining what a child is unable to do, become displaced by a more dynamic view of assessment that uses information about a child's conceptualizations to inform instruction (Jacobs, Lamb & Phillip,2010; Stringer, 2009; Watson, 1996). A pedagogy that sustains assessment as a dynamic process illuminates learners' needs on the basis of actual current knowledge and understanding, rather than on the basis of identifying any gap between where a child is, or should be, within a particular curricular framework (Moscardini, 2014). Supporting the development in practice of this dynamic process was at the heart of our work with the teachers.

Professional Development Model

> *'We have watched many examples of good teachers employing limited methods, that, no matter how competently they are executed, could not lead to high levels of student achievement.' (Stigler & Hiebert, 2009, p.10)*

It is beyond the scope of this report to critique the various and varied initiatives relating to primary mathematics teaching that have swept Scotland in recent years or the professional development models that have sought to implement these initiatives. However, we would argue that in many cases, these initiatives have been introduced in ways that reflect a top-down 'toolkit' approach. They may have become a further addition to a teacher's toolkit in practice, often with weak understanding of, and in some cases subverting, underlying principles but fundamentally they have been mapped on to existing practice rather than leading to a transformation of practice.

Rather than focussing on teachers' competence as a means of improving students' learning, Hiebert and Morris (2012) have presented a powerful argument for focussing on the improvement of teaching and this improvement comes through focussing on children's learning. Their argument connects with Japanese models of teacher development (Lesson Study) that focus on interpreting pupils' learning rather than on techniques of teaching (Watanabe, 2002). This practice has been developed in Europe and in the UK (Dudley, 2012; Norwich & Jones, 2014; Ylonen & Norwich, 2012) but not in relation to mathematics teaching and CGI in Scotland. Research evidence about teachers' learning about mathematics teaching shows effective learning is situated in classroom interactions (Empson & Junk, 2004; Boaler, 2002; Lampert, 2001; Fennema et al., 1996).

The professional development model we worked to on the project was based on the principles of Lesson Study and employed the Purposeful Pedagogy and Discourse Instructional model (Stein et al., 2008) used in Arkansas, USA as part of the Arkansas CCSS Mathematics (Jaslow, 2012). The section below includes explanatory text which has been taken from a briefing paper written for the participating teachers in our project and used in the project workshops.

Lesson Study

Lesson Study is comparatively recent in its development in practice in the UK.

> *'A number of recent, large scale studies have examined the features of professional learning that most impact on classroom practice and pupil learning. These suggest that:*
>
> - *Effective teacher learning takes place over months (not days)*
> - *The classroom is the central location of professional learning activity*
> - *Experimentation or enquiry features in the teacher learning process*
> - *There is collaboration with one or more other professionals as part of that process'*
>
> *(Norwich & Jones, 2012, p. 30)*

Lesson Study involves a process of shared assessment, planning and teaching. It requires deep teacher knowledge and it also supports the development and growth of teacher knowledge. However, in order to support this growth in teachers' knowledge it has been recognised that a critical element is the support of a participant with expert knowledge. Given the acknowledged lack of expertise and depth of knowledge around children's mathematics, the danger is that without this expert support, teachers will simply work within the limits of their existing knowledge base and will be unable to identify that which they do not know (see Hart et al., 2011). A longer term aim of the project was to build this expertise in the schools so that this process could be sustainable. In the short term, it was deemed essential to include expert members of the project team in the lesson study discussions in order to support the teachers.

The research lesson is central to Lesson Study. The research lesson distinguishes Lesson Study from other collaborative observational activities. Its joint planning and collaborative structure means that the focus is on the lesson and the children's learning not on the teacher. It simply means that one member of the planning team is responsible for delivering the lesson but that responsibility is rotated in subsequent lessons. The schools worked to a Lesson Study cycle comprising two to three lessons taking place on consecutive days, which created opportunities for experimentation, reflection and follow up.

The Lesson Study cycle adhered to was as follows:

- Consider goals for student learning and development
- Plan a "research lesson" based on these goals
- Observe the research lesson and collect data on student learning and development
- Use these data to reflect on the lesson and instruction more broadly
  (from Murata, 2011, p.2)

Purposeful pedagogy

The classic Lesson Study cycle used in the project aligned with the Purposeful Pedagogy and Discourse Instructional Model developed by CGI researchers and professional developers (Linda Levi, Jeannie Behrend, Linda Jaslow see Jaslow, 2012) which we drew upon for our project. This model has three components: assessing students, setting learning goals, and designing instruction and draws upon the following elements:

- Types of teachers' mathematical knowledge (Hill, Ball & Schilling, 2008)
- Professional Noticing (Jacobs, Lamb & Philipp, 2010)
- Orchestrating productive mathematical discussions (Smith et al., 2008)

In the workshops with the teachers and particularly through the Lesson Study sessions we focussed on assessment, planning instruction and leading mathematical discussions (Kazemi & Hintz, 2014).

Teachers' mathematical knowledge has been discussed above, below are brief descriptions of the other components. The Lesson Study was intended to provide a framework for discussion and development of these various elements.

Professional noticing

While assessing students, we apply the concept of professional noticing (Jacobs et al.2010).

Professional noticing is comprised of three teacher skills:

- Attending to children's strategies
- Interpreting children's understanding
- Deciding on how to respond on the basis of children's understanding.

Orchestrating productive mathematical discussions

Setting a learning goal and designing instruction
(Stein et al., 2008)

Critical instructional decisions are based on the mathematical interpretation of students understanding. With specialized content knowledge and knowledge of content and students in place, we are ready to focus on our mathematical practice. Elements for the design of the instruction are defined as:

1. Anticipating likely student responses to cognitively demanding mathematical tasks;
2. Monitoring students' responses to the tasks during the explore phase;
3. Selecting particular students to present their mathematical responses during the discuss and summarize phase;
4. Purposefully sequencing the student responses that will be displayed;
5. Helping the class make mathematical connections between different students' responses and between students' responses and the key ideas." (from Jaslow, 2012).

Chapter 4: Making it Happen

Project Delivery

The project comprised an iterative mix of professional development and research.

This section provides an overview of the context, the professional development interventions delivered within the schools the development of materials and processes to support teachers and the research methods used for process and summative evaluation.

Context: The Schools

This report effectively covers three separately funded projects delivered in schools in three local authority areas. The schools were recruited individually but shared a number of common characteristics:

- Each school had identified a need to improve numeracy attainment for pupils
- Each school had at least one member of staff who had completed the University of Strathclyde post-graduate module on CGI
- Headteachers were committed to the introduction of CGI across the whole school, identifying funding from either the Scottish Attainment Challenge or Pupil Equity Fund to support in-school development for teachers
- Each school had a high proportion of pupils living in areas of multiple deprivation

Table 1: Context for Participating Schools

	Hareleeshill Primary School	Bonhill Primary School	St Stephen's Primary School
Local Authority	South Lanarkshire	West Dunbartonshire	North Lanarkshire
Denomination	Non-denominational	Non-denominational	Roman Catholic
School Roll (2017/8)	250	300	240
Staff	1 x Headteacher 2 x Depute Head 2 x Principal Teachers 13 x Teachers 2 x Learning Assistants 8 x ASN Assistants 2 x Early Years Assts	1 x Headteacher 1 x Depute Head 1 x Principal Teacher 15 x Teachers 5 x Learning Assts	1 x Headteacher 2 x Depute Head 1 x Principal Teacher 19 Teachers 2 x Learning Assts 7 x ASN Assistants
Classes	7 x mainstream classes 6 x additional support needs classes 2 x nursery classes	11 x mainstream classes	8 x mainstream classes 6 x additional support needs classes (LCSC)

Taking a Whole School Approach

The research team worked with staff, pupils and parents at each school. Work in the classrooms from Nursery to P7 is detailed in the sections below. In addition, the principal researcher delivered briefings to Classroom Learning Assistants and parents to aid them in supporting pupils.

Each school had a school-based co-ordinator throughout the project. This was a member of teaching staff who was nominated through discussion with the individual schools at the outset of the project. We considered it important that staff should have someone that they could consult with on an ongoing basis. We looked to the most experienced and knowledgeable teachers who had been working with CGI. It was also important that they should feel comfortable in this role and that support and collaboration with colleagues should be viewed with mutual respect. Two of the co-ordinators had limited experience of working with CGI themselves, however it was evident through observation that they were highly effective teachers. One co-ordinator did have over 7 years' experience of CGI. This provided an essential opportunity to support the co-ordinators themselves. We had regular meetings with all three co-ordinators to support them in their role. The development of a sustainable model was as a crucial but longer-term aim of the project. We considered the development and support of individuals who could in time support and lead the development of colleagues within their schools to be a vital component of the project. In the longer term this could be extended to the wider professional learning community through a practice model that would be sustainable and also maintain the integrity of CGI.

The co-ordinators also fulfilled an important practical role throughout the project. They were an important conduit between the teaching staff and the research team and they were able to highlight arising issues and questions. They dealt with all the organizational and logistical matters relating to classroom support sessions, Lesson Study cycles, webinars, professional dialogues and interviews. They also organized their own school-based professional development sessions.

The table below summarises the reach of this professional development programme and research. Over the three-year period of the study team members gathered data in the form of interviews, observations and research notes relating to 75 members of staff. Note that development work began at Hareleeshill Primary School two years prior to the other schools, so staff turnover has been a greater factor here than in the other schools, hence a smaller number of staff completing final interviews.

Table 2: Participation in Research and Development across all schools

	Hareleeshill Primary	Bonhill Primary	St Stephen's Primary
Staff completing Initial interviews	20	15	21
Staff completing Final interviews	14	15	20
Staff participating in at least one Professional development intervention	28/32 individuals on record	17/19 individuals on record	23/24 individuals on record

Professional Development Interventions

Professional development interventions were developed by members of the research team, with input from headteachers, school CGI co-ordinators and individual teachers. Interventions were continuously revised and developed as the project was delivered, in response to issues arising and feedback from teachers and CGI co-ordinators.

Project delivery was complex, requiring significant commitment from CGI co-ordinators; school management teams and individual teachers over one to three years. Given the high level of engagement with classes and teachers within schools – often over short, concentrated periods of delivery such as in-school support from visiting professors – Senior Management support was crucial in ensuring staff cover and time allocation. Individual teachers had to commit time for interviews, study, lesson preparation and feedback over and above their normal workload. For those attending external training there were additional time and travel commitments. The CGI co-ordinators' roles developed from being initial liaison between the schools and research team and became increasingly important in terms of providing ongoing guidance to teachers in school, co-ordinating timetables to ensure cover and availability for classroom observations and dialogue and establishing groups for Lesson Study.

All teaching staff began their CGI learning with either the accredited masters level module delivered at the University of Strathclyde or in-school seminars. Other interventions such as classroom demonstrations and professional dialogue sessions developed iteratively in response to teacher demand and/or review by the research team.

Elements of externally delivered and in-school professional development delivered through the CGI project are summarised in the table below. Throughout the project, professional dialogue is highlighted as critical to success and has been used below to distinguish between elements of professional development. An indication of the professional development, school and teacher resources required to deliver the project is also given.

Table 3: Elements of Professional Development

Professional Development Interventions **External Interventions (out of school)**	**Resource Requirements**
Accredited CPD module University of Strathclyde - Children's development in numeracy: An introduction to Cognitively Guided Instruction - taught post-graduate module (20 Masters credits) includes case study assignment	(not covered by CGI project) 30 hours delivered at University by Dr Lio Moscardini Teacher input: 1-200 hours teacher participation + travel Fees: £450-700 /teacher
Saturday Workshops Stand-alone workshops on children's mathematical understanding/issues raised by teachers/introducing and planning for lesson study.	6 x 3 hour workshops delivered at University by Lio Moscardini, Cheryl Lubinski, Al Otto with contributions from teachers Teacher input: 3 hours teacher participation + travel for each workshop
In-school professional development	
In-service seminars similar to content of Supporting Numeracy Module, without requirement for personal study/assignment/accreditation	~20 hours per school delivered in-school by Lio Moscardini, Effie MacLellan Teacher input: estimated 20 hours in-service
Professional Dialogue (outside the classroom) Small group discussions (e.g. numeracy clinics for teachers); meetings and Skype sessions with research team	Facilitated by Lio Moscardini/Cheryl Lubinski/ Al Otto/Jim Brickwedde Skype/WebEx access when American professors not in the UK Teacher input: estimated 40 minutes/session after school
Classroom Interventions with Dialogue Observations/demonstrations that include dialogue between class teacher and member of research team (typically on professional noticing; mathematical concepts; problem construction)	Facilitated by Lio Moscardini, Cheryl Lubinski, Al Otto, Jim Brickwedde Recorded by Sue Sadler
Classroom Interventions without Dialogue Classroom Demonstration/Observation & support (with no recorded discussion with teacher)	Facilitated by Lio Moscardini, Cheryl Lubinski, Al Otto Recorded by Sue Sadler
Classroom Observations (observations where recorded classroom activity is wholly directed by class teacher and there is no dialogue with research team is regarded as research activity with only incidental impact on professional development, rather than professional development intention)	All members of the research team and some teachers/SMT (within their own school) observed lessons. Classroom observations recorded by Sue Sadler Teacher input: School time allocation/staff cover
CGI Lesson Study Cycles of lesson study incorporating up to 3 lessons planned collaboratively, delivered and observed by small groups of teachers supported to plan and review numeracy lesson, with relevant next lesson planning etc.	Lio Moscardini, Cheryl Lubinski & Al Otto guided discussion on lesson plans; observations; and outcomes with Lesson Study groups Teacher input: School time allocation/staff cover

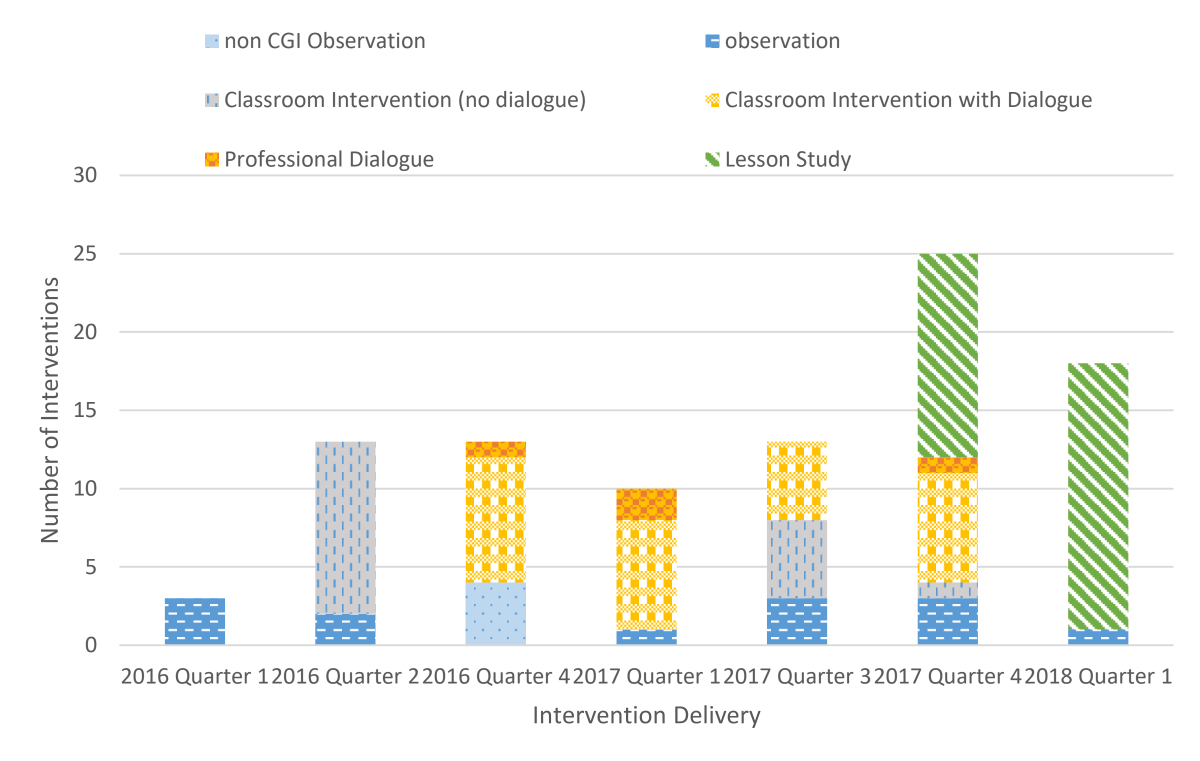


Figure 6: In-School Development with Teachers across all schools 2016-2018

Over the period, the type of intervention changed from observational to increasing level of dialogue about what the research team and teachers saw happening in the classroom. By 2018, development interventions were limited to supporting Lesson Study and Professional Dialogue, and schools had started to organise these sessions independently of the research team.

Teachers from all schools in every year group and composite class, including additional support classes, received at least one observation; classroom intervention or lesson study support from the research team.

Methods Summary

Overall the project engaged with 75 teachers and 790 pupils across three schools. A mix of qualitative and quantitative methods were employed to evaluate project impact on pupil attainment and teaching practice. Research opportunities such as classroom observations were built in to professional development interventions wherever possible to establish the impact they were having in the classroom and guide development of interventions.

The research methods chosen to identify change are summarised in Table 4 below.

Ethics

The project conformed to the requirements of the University of Strathclyde Ethics Committee. Consent for participation was approved by Local Authorities. Participants were provided with information sheets outlining the details of the project and the nature of their participation. Each participant completed and returned an individual consent form.

Table 4: Research Methods used to Measure Change in Pupil Attainment and Teaching Practice

	Measures	Tools
Pupil Attainment	Year on year change in individual score on CGI test	CGI Assessment developed for the project (see p.40)
Pupil Attainment	Year on year change in individual score on standardised numeracy test	GL Assessments CEM Assessment CfE Assessments Learning Community Assessments
Professional Development	Year on year change in teachers' views on their preparedness to teach mathematics	Interviews
Teacher knowledge & beliefs	Year on year change in teachers' knowledge and beliefs about learning and teaching mathematics	Interviews Classroom observations
Elements of Instruction	Change in content and delivery of numeracy lessons over the course of the project	Interviews Classroom Observations
Other – inclusion/impact	Year on year change in inclusion, pupil outcomes and classroom impact	Interviews

While the three schools engaged with the project individually, and differences were noted in levels of engagement with project interventions (particularly among Senior Management Teams), to safeguard the anonymity of participants, both quantitative and qualitative data have been analysed as a single data set. This approach has the added benefit of reducing the effect of contextual factors within individual schools allowing us to focus on change in numeracy teaching and in pupil attainment, as well as the links between the two.

Overall, 488 pupils undertook CGI assessments in the final year of the project. From these, we were able to identify 333 matched pre- and post-intervention assessments that were analysed using a statistical software package to identify changes in attainment and whether such changes were statistically significant. The research team undertook 105 interviews and 95 classroom observations over the period of the project. Figure 7 below shows when these research tools were deployed.

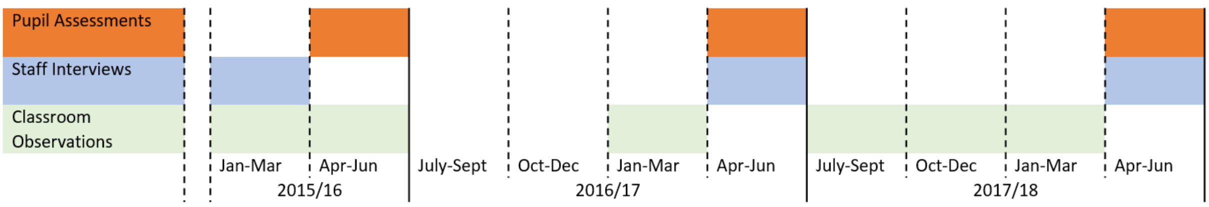


Figure 7: Application of Research Tools across the Project Timeline

Quantitative Data and Analysis

An assessment tool was specifically designed for the project. It was based on materials originally developed by Professor Elham Kazemi and Kendra Lomax of University of Washington, USA with additional materials developed by Dr Penny Munn and Dr Lio Moscardini at the University of Strathclyde; it was then piloted and adapted for the project by Chiara Moscardini McKenna. These assessments were used with the developers' expressed permission.

Reflective of what Skemp described in his seminal paper as instrumental and relational understanding (Skemp, 1976), the assessments were designed to highlight any progression in children's understanding and mathematical thinking and provide teachers with insight into children's conceptual understanding rather than focus only on procedural competency. It was with this in mind that the assessment tool presented students with the same types of problems at both pre and post assessments. This allowed us to see not only the particular strategy which a child might use to solve a problem but also the conceptual sophistication of the child's solution strategy in relation to a particular problem type. Pupils' assessments were coded to reflect the actual strategy used as well as the conceptual sophistication of the strategy in relation to the problem type.

We spent time in each school explaining the assessment tool and the process. Teachers were provided with guidelines, extracts of which are replicated in Chapter 6 below.

Pre- and post- intervention assessments

We assessed as many children as was practical in each school from the outset of the project. The intention was to assess every child and in Hareleeshill Primary every child was assessed at the outset and in the other schools nearly every child was assessed. However, the final assessments coincided with the introduction of the SNSAs which along with other pressures which teachers were under meant that they were unable to assess every child. There was no selection or exclusion criteria applied to who should be assessed so in this sense the process was random. The statistical analysis of the assessments has only been applied to cases where we had both pre and post assessments.

We assessed 488 (62%) of all pupils in the schools where the interventions took place. 479 pre-tests were conducted, and we were able to match pre- and post-test data for 333 pupils.

Table 5: Pupil Assessments across the three schools

Schools 1-3	number
Total pupils	790
Total pupil CGI assessments	488
Total matched pre & post assessments	**333**
Primary 1	61
Primary 2	55
Primary 3	60
Primary 4	41
Primary 5	52
Primary 6	45
Primary 7	19

An important point to note in the assessment process is that in the second assessment children were given the same problem types as in the first assessment. This was to allow us to see if children were applying more sophisticated solution strategies and demonstrating greater conceptual understanding. We developed a coding frame for the statistical analysis (SPSS) that considered both the specific solution strategy applied by the pupil and conceptual sophistication.

The coding of solution strategies from 0 to 18 is essentially descriptive and reflects a hierarchy of strategies. A pupil in Primary 1 at pre-test tackling a Join Change Unknown problem by direct modelling can be considered to have made progress at post-test if their strategy has moved to counting on.

However, we would not consider that direct modelling is as sophisticated or appropriate for a child in Primary 6 tackling a multi-digit Join Change Unknown (JCU) problem. Scoring conceptual sophistication on a scale of 0 to 10 enables us to take account of how appropriate the strategy is in the context of the problem and the age of the pupil. Results are therefore reported in terms of the conceptual sophistication of solution strategies deployed.

Table 6: Coding for Solution Strategy and Conceptual Sophistication

Solution Strategy	Code	Score for Conceptual Sophistication
Absent	0	0
Not Attempted	1	0
Invalid Strategy	2	1
Trial and error	3	1
Valid strategy - unsuccessful	4	2
Direct modelling	5	3
Counting all	6	3
Skip counting	7	6
Counting on from first number	8	4
Counting on from larger number	9	5
Counting on from	10	6
Counting on to	11	7
Counting down from	12	7
Counting down to	13	8
Derived facts	14	9
Direct recall/known fact	15	9
Invented algorithm	16	9
Relational working	17	10
Standard Algorithm	18	7

Qualitative Data and Analysis

Qualitative data from staff interviews have been matched – using only data where we have pre- and post- intervention records, together with two or more classroom observations and at least one lesson study, giving in depth qualitative data on 17 teachers across P1-P7. Hereafter this data set is referred to as 'core' data. Other interview data was examined to establish the range of opinions.

Table 7: Composition of Core Interview Sample

	Number of teachers Nursery- P3	Number of teachers P4-7	Teaching experience	CGI Accredited Module	Total teachers in core sample
School 1	1	2	2-7 years	0	3
School 2	6	3	4-36 years	5	9
School 3	2	3	1-28 years	0	5
includes 2 ASN teachers					**17**

The core interview data were analysed using a Framework approach (Ritchie, et al 2014). Themes identified in earlier research (Moscardini, 2014) were used to initially classify the data and a number of additional themes emerged (marked with an asterisk in the table below). Data was then summarised and examined for evidence of change. All interview quotations are extracts from the core data set unless otherwise specified.

Table 8: Thematic Framework for Interview Data

Theme	Theme
Elements of Instruction	**Professional Development**
Assessment	Future PD*
Autonomy	PD- Attitude to self
Determinant	PD- Issues
Determining Strategies	
Didactics	
Pedagogy	**Other**
Resources	Classroom Impact of CGI*
Role of Teacher	Risk*
Knowledge & Beliefs	Future practice*
Beliefs about being numerate	Inclusion
Beliefs about pupil learning	Pupil Outcomes*
Beliefs about teaching	
Eureka moments	
Knowledge of learning	
Pedagogical knowledge	
Subject knowledge	

Classroom observations were summarised using 16 themes broadly covering class and date; whether CGI was being used with whole or part of the class; the problem type(s) set and rationale; solution strategies; mathematical focus and discussion; pupil/teacher roles; use of materials; inclusion and pupil engagement.

Table 9: Thematic Framework for Classroom Observation Data

Name
Period (relevant year & quarter)
Class
Whole or part class CGI
Problem type(s) set
Rationale for problem selection
Mathematical focus of the session
Mathematical principles made explicit
Role of teacher / pupil autonomy to problem solve
Use of materials by pupils
Presenting/Sharing solution strategies
Professional noticing & recording
Issues of inclusion/equity
Teacher observations/ remarks
Pupil engagement in mathematical activity
Making connections (with mathematics or strategies)

Throughout the project members of the research team dynamically assessed changes in teachers' knowledge and beliefs at each school using Fennema et al. (1996). This framework classifies teachers' CGI practice at four levels:

Level 1: Teachers believe that they need to demonstrate procedures to pupils and classes are dominated by repeated practice of standard procedures.

Level 2: Teachers begin to question whether pupils need explicit instruction and start to provide opportunities for pupils to use their own strategies.

Level 3: Teachers believe that pupils can solve problems without explicit instruction. Pupils solving problems and discussing solution strategies dominate classroom activity. Teachers set appropriate problems, ask questions to understand children's thinking and appreciate the variety of solution strategies their pupils devise.

Level 4: Teachers' understanding moves to a more conceptual level at which teaching is driven by pupils' thinking and teachers become more independent learners about children's mathematical thinking.

Chapter 5: Developing Professional Learning for the Project

The above figure 6 shows a move toward an increasingly dialogic approach as the project developed, with Lesson Study taking over as the preferred form of intervention by 2018. The sections below provide a brief description of the elements of professional development identified in the above summary, an indication of when they were delivered and to whom, highlighting perceived strengths and weaknesses in design/delivery and consequent developments in design.

Accredited Professional Development

Children's Development in Numeracy: An introduction to Cognitively Guided Instruction –was a taught post-graduate module accredited at Masters level (SCQF 11) developed and delivered by Dr Lio Moscardini. The module ran in September/October. Teachers from each of the three schools successfully completed the module which also involved a classroom-based assignment.

Table 10: Delivery of Accredited Professional Development

Academic year / Schools	2014/5	2015/6	2016/7	Total teachers trained/total employed
Hareleeshill	1	7	4	12/18
St Stephen's			7	7/23
Bonhill			1	1/18

The module was designed to provide:

- Background pedagogical and subject content knowledge
- Opportunities for reflection and dialogue with experts in the field and other teachers
- A bridge from theory to practice through an in depth classroom assignment

The specified aims of the course were:

- To develop teachers' understanding of children's mathematical development so that they are equipped to recognise and interpret children's mathematical understanding and support **all** children in this area.
- To apply their knowledge of children's mathematical development and processes of educational support within the context of an inclusive pedagogy which focuses on ways of teaching that are common to all learners and is also responsive to individual needs.

The module did not run in 2017/18.

Contribution to Developing Practice of CGI

This form of professional development outside the school and pressures of the working day allows teachers to invest their own time in learning and reading about the background and evidence for CGI that underpins the changes in practice that are required to teach in this way. The class was theoretically driven and rooted in practice; teachers were required to develop and report on an individual case study in relation to their own practice.

> *"when I was doing my own case study for the course, it was probably one of those moments when I was working with the wee boy that my study was on, and I found that he could actually do a lot more than I'd ever given him credit for."*
> Early Years Teacher with 10 years' experience

Evidence of its contribution to improved classroom practice comes from qualitative feedback from participants. The Headteacher at Hareleeshill Primary was sufficiently inspired by her own experience to argue for participation by other teachers at the school and for the school to work with Dr Moscardini to improve mathematics teaching at the school through what became the SAC CGI project. Equally, other staff members were motivated to take the course by what they saw and heard of the Headteacher's experience.

One of the teachers, who later participated, commented at interview that,

> *"The CGI course is by far the best thing I've ever done in my teaching career and I think people who haven't done it – it's a MUST. It's taught me so much. I feel like my eyes have been opened and you can see things that you couldn't see before."*
> ASN Teacher with 8 years' experience

At Bonhill Primary, one teacher had undertaken the class as part of her own professional learning. The pupil assessment results for a group of low-achieving children in her class improved so strikingly after the teacher completed the module that the Headteacher sought to join the CGI project.

St Stephen's Primary involvement came about following a visit by the Headteacher to Hareleeshill Primary in 2016. Inspired by what she was observing in classrooms she enrolled herself, two Deputy Headteachers and five class teachers onto the class for session 2016/17.

Limitations

The financial and time requirements of the module are relatively high, with most of the financial burden falling to schools and the time burden falling to individual teachers. Not all schools have been able to invest as heavily in accredited CPD and it would be unrealistic to expect all teachers within a school to undertake the module, given all the other demands on teachers and staff turnover. Although the class offered a good opportunity for collaboration and sharing of ideas with teachers from a broad range of establishments, it was not feasible to provide on-going follow support to individual teachers or schools to help sustain the development in practice. Consequently there is a danger that the initial development will only occur at a superficial or theoretical level which in practice will wash-out or potentially lose its integrity.

In service seminars

In-service seminars were made available to all the schools. These seminars comprised the core content from the certified module and were intended as a first introduction to CGI for teachers' own practice, particularly for those who had not participated in the accredited course. Module participants were encouraged to participate and share their experiences so far.

Table 11: Participation in In-service Seminars

Schools	2016/7	2017/8	Participating Teachers
Hareleeshill	Counting – 1 day, August '16		13
St Stephen's	Intro to CGI – 1 day June '17	Intro to CGI – 2 days Aug '17	17
Bonhill	Intro to CGI – 2 days June '17	Intro to CGI – 1 day Aug '17	15

Contributions to Developing CGI Practice

The sessions were delivered to staff within the same school and so they provided a shared experience and a platform for discussions among the teachers. They were more accessible than accredited professional development given the lower costs, less travel and less teacher time commitment. They also provided an opportunity for all teaching staff to take part and in some cases this included pupil support staff.

While the in-service seminars were initially intended as a stand-alone introduction to CGI for teachers who had no other exposure, they also became opportunities for some teachers who had undertaken the module to engage in professional dialogue with their colleagues and to share their experience of CGI in the classroom. This enabled and reinforced the 'whole school' approach. The majority of Hareleeshill Primary School staff had already completed the introduction to CGI class and their in-service session focussed in depth on children's counting. Evidence of its contribution to improved classroom practice comes from qualitative feedback from participants in the 'Counting' seminars delivered by Professor Effie Maclellan.

> *"The lecture that Effie gave us, …. that effected how I taught quite profoundly, because I've been doing a lot of counting and really honing in on how they count and what to give them next"*
>
> Primary 2 Teacher with 14 years' experience

The introductory sessions in St Stephen's Primary and Bonhill Primary were intentionally spread out so that teachers could have time to apply some of their learning in classrooms and share their observations at subsequent sessions. One of the benefits of this approach was that it connected theory to practice in a way that supported the development of practice through cycles of engagement and reflection.

Limitations

The relatively limited time commitment associated with in-service seminars and the almost immediate introduction thereafter of CGI in the classroom meant that teachers had less opportunity to do additional reading and deepen their theoretical knowledge. Without the practice-based case study assignment which was a requirement of the accredited class, teachers may spend less time focusing on individual students perhaps moving straight in to managing a full class before sufficiently developing their own practice in understanding children's mathematical thinking and recognising pupil solution strategies. This issue was recognised by the headteachers in the schools, one of whom specifically suggested that any future school-based professional development should include a case-study element. Supporting teachers in practice through embedded classroom activities would help bridge between theory and classroom practice.

There may be a concern that in-service professional development is less valuable for teachers' CVs when changing jobs than accredited professional development. Nonetheless, in terms of raising awareness of CGI as an approach to numeracy education, the Principal Investigator has already in the course of the project, been contacted by a school who had heard of CGI through a supply teacher who had recommended CGI, having seen it in action at one of the participating schools. He has also experience of professional learning being delivered by teachers with little and in some cases no prior professional development in CGI or experience of working with CGI in practice. The danger here is that some teachers may have experienced professional development in CGI that may be of dubious quality with the consequence that it is misrepresented and dismissed as ineffective.

While the 'whole school' approach is conceived as establishing a supportive environment for teachers introducing CGI, an element of coercion may be perceived among some teachers who may be fearful or reluctant to change their practice. At the very least, there will be a range of attitudes among teachers toward embracing CGI. Equally, within each school teachers will have a range of mathematics knowledge and understanding that can affect both their ability and confidence in delivering CGI in the classroom.

Saturday Workshops

A series of stand-alone workshops were held on Saturday mornings at the University of Strathclyde. Workshops were led by Lio Moscardini and Cheryl Lubinski with contributions from Al Otto on mathematics and from teachers in the project schools on classroom practice. These workshops were not initially part of the project plan, but emerged in response to pressures perceived by the project team in terms of both the teachers' subject and pedagogical content knowledge. The school-based co-ordinators, in consultation with their colleagues, were key people in identifying and recommending themes that the workshops could usefully focus on.

Some teachers appeared to lack a conceptual understanding of the mathematics they were teaching, resulting in a reliance on procedures and difficulties in assessing the full range of strategies used by their pupils. Workshop activities focussed on developing teachers' understanding of children's solution strategies and connecting this with the teachers' own mathematical understanding.

Teachers, particularly at the upper primary levels, needed more support in linking CGI to Curriculum for Excellence. Sessions that focussed on this were led by one of the more experienced school-based co-ordinators, Chiara Moscardini McKenna, who has been working with CGI in practice for nearly eight years. The feedback from the teachers was that it was extremely useful in helping them deal with issues relating to planning and transitioning.

The workshops provided an invaluable opportunity for the teachers to share their experiences across the three participating schools. Teachers came with examples and questions relating to the children in their classes. A question which is asked frequently when teachers begin implementing CGI in practice is 'where do I go next?'. Through practical discussion based on real examples, the workshops provided an opportunity for teachers to consider this and supported them in identifying appropriate next steps.

Table 12: Saturday Workshops

	2017	2018
Number of Saturday workshops held	2 x September 3 x November	1 x March

Participation in the Saturday workshops averaged 22. Teacher participation in Saturday morning workshops in central Glasgow demonstrate considerable commitment by teachers and eleven teachers reported attending all six workshops, with an additional 31 teachers attending at least one workshop.

In School Professional Development Interventions

Professional Dialogue

There were a range of interventions based around professional dialogue. A key element of this was the international videoconferencing (VC) sessions between school staff (most of whom were expected to participate) and research team members from the US and in-school dialogue.

These sessions were conceived as opportunities to benefit from the experience of US professors through videoconferencing. Initial whole school skype sessions between Hareleeshill (with Lio attending at Hareleeshill) and Cheryl Lubinski/Al Otto in the US were very well attended and ran with very open agendas. Teachers were alerted when the date and time of the skype session was set that this was an opportunity for them to raise questions or to discuss examples of practice from their classrooms. Technically the sessions depended on the availability of hardware and the goodwill and IT experience of staff in the school.

When Lio Moscardini and the US professors visited Hareleeshill Primary in the early stages of the project (and initially at Bonhill), they held similar whole school open discussion sessions. Over time as the project (and teachers' knowledge) grew, the balance shifted from whole school discussions, to smaller groups, developing other opportunities for 'professional dialogue', for example, following classroom observations.

Bonhill Primary very quickly (in the first term of the CGI project) adopted separate discussion groups for upper and lower primary teachers.

This was occurring in professional development delivery at the same time as the technical issues involved in video conferencing became more challenging. Local Authorities appeared to be tightening up on the connectivity and security of the schools' systems and videoconferencing through skype (with which parties were familiar) was no longer available. The frequency of such general video conference sessions reduced in 2017, as did the frequency of general, whole school Q&A sessions.

The agenda for sessions that took place were increasingly planned in a collaborative way. This was more effective for the US professors, allowing them to prepare and consider classroom examples, and was made possible by the growing knowledge and experience of CGI among participating teachers. It also had the value that teachers were better placed to determine whether particular sessions were likely to be of benefit to them.

By the time video conference sessions using WebEx were planned in 2018 with Jim Brickwedde, the schools were structuring separate sessions for upper and lower schools, with the school CGI co-ordinators taking over the lead role in planning and scheduling from the research team.

Table 13: International videoconferences & In-school Sessions

	2016	2017	2018
General Q&A sessions (VC sessions)	Hareleeshill, 11 May LM (CL,AO) Hareleeshill, 25 May LM (CL,AO)	Hareleeshill, 22 Feb LM (CL,AO)	Bonhill, February, (JB) St Stephen's, February, (JB) Hareleeshill, February, (JB)
General Q&A sessions	Hareleeshill, 15 June LM,EM, CL,AO Hareleeshill, 1 June LM, CL, AO, JB Hareleeshill, 28 November LM,CL,AO		LM- Lio Moscardini CL - Cheryl Lubinski AL- Al Otto JB - Jim Brickwedde

Classroom Interventions without dialogue

Working alongside teachers in the classroom Cheryl Lubinski gave a number of demonstration classes that involved whole classes working with CGI. These lessons emerged from opportunities to more fully develop the pedagogy, to explore aspects of mathematics, or at the request of teachers seeking guidance on the introduction of a new concept, such as fractions. Opportunities for dialogue were very limited and none were recorded.

'Class support' was a slightly modified version of the demonstration class, in which Lio or Cheryl would work alongside the teacher following the teacher's original lesson plan but if appropriate would intervene to highlight aspects of CGI, for example how all pupils could engage in just one problem and the mathematics that could be drawn from it.

Over time, classroom demonstrations and class support gave way to classroom observations, professional dialogue and lesson study (see below).

Contribution to developing CGI practice

"I think at the start I just thought it was problem solving and that's it, and that you don't actually do any teaching, but from observations and seeing Lio and Cheryl, it made me realise, well, grab those teachable moments, and that's your time to go, "Well, what you, Child-A actually did there, was this", which is good because it's a bit more organic, rather than standing at the board and saying, "this is the way". "

Primary 3/4 Teacher with 7 years' experience

Prior to the project, teachers had no real-life exposure to CGI delivery in the classroom. The CPD sessions and reading prepared teachers to some degree for a radically different classroom. There was little guidance on how to transition from a traditional procedure-based learning environment with its emphasis on independent working assessed on getting the right answer, to a CGI classroom that encourages pupils to use a wide range of materials to develop their own solution strategies and stimulates dialogue about mathematics. Some teachers actively requested to know what this would look like in practice. Classroom demonstrations and class support helped to fill this gap.

Classroom demonstrations and class support also provided opportunities for teachers to concentrate on how pupils are learning without worrying about how their own delivery might influence the pupils' experience.

In the early phase of the project, teachers introducing CGI at Hareleeshill initially worked with very small groups of often struggling pupils within the class. It is easier for teachers to build their confidence and skills at setting word problems, recognising and supporting different solution strategies by working 1:1 or in very small groups. A number of staff expressed concerns about classroom management in introducing CGI to whole classes, although this varied from school to school and class to class. Demonstrations might appear a useful response to such concerns, however there was no explicit recognition of this in teacher interviews or storyboard.

Limitations

The main limitation of this approach, and we consider it to be a significant one, is that it allows teachers to take a back seat rather than developing their own skills by delivering CGI lessons and can be used to avoid introducing CGI to their own practice.

It was challenging for the research team to provide class support following a lesson plan devised by the teacher without prior consultation. At times the mathematics emerging from the problem(s) set by the teacher were not what the teacher had intended and with no engagement in planning the lesson, it was a struggle then to achieve the lesson goals.

"I've given them multiplying decimals, they've never done that before, and obviously the answer gets smaller rather than bigger. And that's what I did when Cheryl and Al came in, and it didn't go very well. Cheryl and Al were struggling then to see how to explain it to children and how they would go forward with that."

Primary 7 Teacher with 5 years' experience

Although, particularly at the early stages of the project, there were regular requests from teachers for classroom demonstrations 'to help us see what it looks like' there are several problems with this approach. As stated, it risks positioning teachers as passive observers who should replicate what they see. There is a risk that teachers will think that there is a specific and singular model of practice that should be adhered to and most significantly, it hinders teachers from actively engaging in the learning and teaching process. While we were eager to support teachers and help them feel confident, we did not consider this to be a helpful model and we were keen to move towards a more embedded model of practice and support which is represented by the Lesson Study model.

Classroom Observation with Professional Dialogue

As teachers began implementing CGI in the classroom, it became increasingly relevant to focus discussions on specific issues, whether subject or pedagogy based. In addition, classroom observations also generated topics arising from shared experience of the classroom. Given the challenges of attempting to hold such discussion in the classroom, these professional conversations generally took place with small numbers of teachers to whom the topic was of specific interest; or as follow up meetings held as soon as possible after a shared classroom experience. Hareleeshill Primary School also adopted a 'clinic' approach to numeracy, directed and delivered by the CGI co-ordinator there independently and in addition to the CGI project. It should be noted that the term 'clinic' was chosen by the school. The model of this clinic was based on supporting teachers' learning and only involved teachers. These clinics provided opportunities for teachers to share and discuss practice and also readings about children's mathematics. They bore no resemblance to other curricular clinic models which reflect a medical model where children are extracted from classrooms for intervention from students.

The classroom observation and dialogue approach developed into CGI Lesson Study (see below).

Professional dialogue

The value of setting aside even 10-15 minutes of teacher time with the CGI project team members who had observed classes was increasingly recognised as the project progressed. One advantage of continuing such professional dialogue outwith the classroom was that it allowed both project team members and teachers to reflect on what they had seen and for the project team to reflect on resources that might be useful for the teachers.

Contribution to Developing Practice of CGI

Seeing teachers delivering CGI in the classroom was immensely useful. It allowed the CGI project team to benefit from the teachers' knowledge of the individual pupils in their class and to share the precise interaction between pupil and teacher that led pupils to address problems in particular ways. During the class, members of the team could flag up particular strategies being used and ask questions that would assist teachers to plan plenary sessions where pupils shared their strategies with one another. The short follow up sessions supported teachers in interpreting the strategies they saw pupils using and planning best next steps to stretch or consolidate learning.

Limitations

The teacher's learning intentions and drivers for the choice of problems was not always obvious to the CGI project team. Moving to a Lesson Study approach addressed this point.

CGI Lesson Study

One of the Saturday workshops (November 2017) was used to introduce the concept and planning tools for CGI Lesson Study. In this model of lesson study small groups of teachers (ideally at least 3) work together to plan, deliver/observe and reflect on their interpretations of children's thinking and how to use that to inform teaching (*professional noticing*, see Jacob et al., 2010), with the objective of assessing pupil learning and designing progressive instruction. Specific frameworks and recording tools were developed to support this process. This lesson study model developed from a combination of lessons learned from the earlier stages in the project, background research and practice observed by Lio Moscardini in schools in Arkansas in December 2017.

Research elsewhere has identified features of professional learning that most impact on classroom practice and pupil learning, highlighting the importance of situated learning in the classroom, collaboration with other professionals, timescale, experimentation and enquiry (Norwich & Jones, 2012, p. 30).

Contribution to developing CGI practice

Lesson Study is a major step forward in preparing schools to sustain and develop CGI practice with fewer external resources. It embeds pupil assessment, lesson planning, professional noticing and professional dialogue. It also provides valuable records for teachers and the project team to focus their efforts in determining the most effective means of guiding pupils to the next stage of mathematical understanding. It is anticipated that reaching this shared level of understanding of CGI delivery will enable external project input to reduce in intensity, to be replaced by wider collegiate discussions in school and beyond, including use of the CGI Facebook group. The Lesson Study model allowed schools to take ownership and build on existing practice. It fostered a strong collegiate approach and there was evidence that it was influencing practice beyond CGI with two schools reporting that would use the model to support the development of other curricular areas within the school.

Limitations

Teachers need support to enable them to find time to work in small teams, for planning, delivery and review. They need to be comfortable working together and delivering a class alongside their colleagues, and in reviewing the class in terms of pupil learning resulting from the team devised lesson plan rather than individual lesson delivery. The involvement of a member of the Lesson Study team who was knowledgeable and experienced in CGI was recognised as crucial. The model supported the development of this level of expertise with an expectation that the school-based co-ordinators in time could assume this role. The absence of such expertise risks a replication of weak and sometimes ineffective practice that is not questioned; this weak model of practice has been critiqued earlier in this report.

Chapter 6: Project Outcomes

Introduction

The project's theory of change proposed that investment in teacher professional development would lead to changes in classroom practice and improvement in pupil attainment. This chapter addresses questions about what changed following professional development interventions and the effect on pupil attainment.

Pupil attainment is addressed in the first part of this chapter, using statistical analysis to explore attainment over the school year 2017/8, supplemented by teachers' views about what made a difference and examples of pupils' work from classroom observations, interviews and fieldnotes that give insights into some of the underlying processes which contributed to these outcomes.

Using assessment tools developed for the project it has been possible to identify changes in pupils' knowledge, strategies used in problem solving and conceptual understanding, to answer the following research questions:

> RQ 1 Following professional development in CGI, is there evidence of an increase in attainment for children across the primary school?
>
> RQ 4 Is there evidence of growth in children's conceptual understanding in numeracy?

The second part of this chapter uses qualitative evidence from interviews and classroom observations to address the remaining research questions about the relationships between teachers' knowledge; classroom practice; pupil engagement and inclusion.

> RQ 2 Is there evidence of growth in teachers' knowledge of children mathematical thinking following professional development in CGI?
>
> RQ 3 What factors contributed to any growth (if any) in teachers' knowledge of children's mathematical thinking?
>
> RQ 5 What is the nature of any relationship between children's engagement in mathematical activity and teachers' pedagogical practices?
>
> RQ 6 To what extent, if at all, are the affordances of CGI responsive to issues relating to inclusion and equity?

Pupil Attainment

Statistical analyses have been undertaken across classes and using items from the assessment tool described below that have been consistently measured at Pre-test and Post-test across all schools. This section presents thinking behind the assessment design, the key areas covered, findings on pupil attainment and related extracts from the qualitative data that offer insights into classroom practice and teachers' views.

Assessment Design

The problems in the assessments were designed to show pupils' thinking and understanding in key areas of numeracy. The assessment typically took 20-30 minutes per child or small group. The earlier stage assessments which involved counting activities were required to be administered on an individual basis. It was emphasised that teachers should use their judgment in selecting assessments that would be appropriate to individual children as opposed to going only by chronological age or year group. This was particularly relevant when assessing children with additional support needs.

The Primary 1-3 assessments consisted of two parts:

Part 1- Counting, Number Recognition and Number Sequence

Part 2- Word Problems

The assessment for Primary 4-7 contained only word problems and equations. At these stages the assessment could be given to small groups of around four pupils and the teacher could take notes and ask questions as they work independently.

Teachers were provided with an assessment script which included a guide on how teachers may wish to introduce the assessment. The assessment followed a dynamic process in which the assessor might seek to elicit a child's thinking but should not prompt a child to a solution. An extract from this guidance is below:

As children are attempting problems, observe what they are doing for each problem and make notes on the detached front sheet. Encourage pupils to tell you about their thinking, even if it is incomplete, or write down their ideas or what they are confused about. This will help you decide what makes sense to them and how they are thinking about the problem. You might ask questions to elicit children's thinking and gain a better picture of their understanding.

Children should not be shown or instructed how to solve the problems or told which strategies to use. The focus of the assessment is on the child's thinking and ideas.

The problem can be read to pupils as many times as necessary. We want to understand how pupils think about the problem – it is ok to help pupils understand what the problem says, but don't prompt their thinking about how to solve it. For example, "Do you know what a pack of pencils might look like?" can help a child to access the story, but "Her mum gave her some more, does that mean the answer gets bigger?" implies an action or strategy for solving. It is okay to repeat the problem or repeat parts of the problem. If a child is unable to address a problem, make a note if possible of what the child has attempted or at which point they got stuck but it is ok to abandon the problem if necessary. *CGI Assessment Guide (2017)*

Key Assessment Areas

The key areas covered across the various assessments are as follows:

Counting and Number Recognition and Sequence Tasks

Addition and subtraction (joining and separating problems)

- Join Result Unknown (JRU)
- Join Change Unknown (JCU)
- Separate Start Unknown (SSU)

Multiplication

Division (partitive)[1]

Fractions

Open Number Sentence

Relational thinking problem

Children's solution strategies and strategy sophistication

Teachers had learned about children's solution strategies in some depth in the various professional development sessions described above. These are described more fully on page 9 of this report, briefly, they include: direct modelling, counting, derived facts and direct recall.   Teachers were required to record children's strategies and were given the following guidance:

> *Look for the type of strategy that pupils use and make notes on the back of the detached cover sheet, beneath the relevant problem. The child's strategy will likely fall into the general category of modelling, counting, derived or known facts or invented or taught algorithm. If a pupil uses a standard taught algorithm as their first strategy, ask the pupil if they can show their thinking in another way or if they can explain their work. This will give you a better idea about whether they are using it procedurally or if they have a deeper conceptual understanding.*
>
> *If pupils use a valid strategy but make an error, make note of what the error was. Pupils might miscount objects in modelling a large quantity, skip numbers in the counting sequence, lose track of a multi-step strategy, among other errors. Making note of this will give a clearer picture of the pupil's thinking.*
> *CGI Assessment Guide (2017)*

We assessed 488 (62%) of all pupils in the schools where the interventions took place. 479 pre-tests were conducted, and we were able to match pre- and post-test data for 333 pupils.

[1] Division can be either partitive (sharing) or quotative (measurement). This is an important distinction which is reflected in children's solution strategies. It is critical that teachers understand this distinction and how this relates to supporting understanding in key areas such as place value and fractions. Very few of the participating teachers were aware of this distinction prior to professional development at the outset of the project.

Other assessments

Each of the schools used had its own assessment for numeracy in accordance with practice in the particular Local Authority. The timing of our project coincided with the introduction of the Scottish National Standardised Assessments (SNSAs). This meant that towards the end of the project the schools had carried out SNSA assessments with the relevant year groups but as this was the first time that these assessments had been administered we have no means of comparison in relation to previous year groups.

We have sought, as far as possible, in this report to maintain anonymity of the participating schools in relation to specific practices and have only provided contextual information where relevant and appropriate. For this reason, we will outline the various assessment tools used by the schools but without specifying particular schools. One school made use of GL assessments, one school had used CEM assessments and one school used an assessment tool developed and used within its Learning Community. This meant that results could not be consistently compared across all schools. Also because of the introduction of SNSA assessments some of the schools did not continue with the previous assessment tool.

For reasons stated, these assessments gave an incomplete picture but there were two salient features worth commenting on. In the school which used GL assessment, which is a standardised assessment, it was administered across the whole school prior to any CGI based activity and then again towards the end of the academic year. The results showed an improvement across the whole school with all but one child in the school showing a rise in attainment; quite a number of children increased by three or four stanines. Results for the first school with which we worked in relation to teacher judgement of CfE levels, showed a large rise in attainment in P4 and P7 classes which had been working with CGI.

Table 14: Percentage children achieving CfE Level (numeracy) relevant to their stage

Stage	2015/16	2016/7
P1	80% - <90%	70% - <80%
P4	10% - <20%	50% - <60%
P7	<10%	70% - <80%
combined	30%-<40%	60% - <70%

The SNSA results for all three schools were also considered to be positive. While these results needs to be treated with caution in respect of attributing such outcomes exclusively to CGI, there was a general indication, supported by comments from the headteachers and teachers from all three schools, of things moving in a positive direction. The results from the CGI assessments which were used by all of the schools present a more robust and quantified account of pupils' attainment.

Pupil Attainment Results

This section looks at the statistical evidence of change in assessment scores together with examples of pupils' work and teacher views of what made a difference. Results, examples and views are presented for Primary 1 & 2; Primary 3 and Primary 4-7. The qualitative data which follow the presentation of the quantitative findings relating to the various year groups, are taken from classroom observations, interviews and fieldnotes. These data provide some insight into the teacher moves that contributed to the outcomes.

Primary 1 & 2 Attainment

The Statistical Evidence

Attainment for pupils in Primary 1 and Primary 2 was measured in the same way, so results were combined to give a larger sample size for analysis. Results suggest that there is a statistically significant increase in the proportion of pupils identified as 'secure' on a range of mathematical questions from pre-test to post-test.

Table 15: Changes in the proportion of Primary 1 & 2 pupils performing securely over time

Mathematical Domain	School 1 to 3 (n=116)		
	P1&2 Pre - test % secure	P1&2 Post - test % secure	% Difference
Cardinality	81.9	97.4	15.5***
Correspondence	83.6	95.7	12.1**
Number Sequence	63.8	94.8	31***
Forward Number Sequence	46.6	89.7	43.1***
Number word After	76.6	97.4	20.8***
Number ID	88.8	97.4	8.6**
Backward Number Sequence	87.1	97.4	10.3**
Number Word Before	63.8	94.8	31***

NB: Statistical test of significance undertaken using McNemar's Chi-Square test.

***p<.001; **p<.01, *p<.05

Result from McNemar's Chi-square test (Table 15) above suggest a significant proportion of P1 & P2 students became more secure in their ability to solve domain specific mathematical questions after the CGI intervention. However, we cannot rule out maturation and cannot tell what proportion of change is due to the intervention.

We also sought to know more about changes in pupils' mathematical thinking. The assessments were designed to identify the solutions strategies used by pupils and the conceptual sophistication of that strategy (see p.25 for details). Conceptual sophistication of strategy was measured on a scale of 0 to 10.

Table 16: Primary 1 & 2 Conceptual Sophistication

	P1 & 2 pre test average	P1 & 2 post test average
Join result unknown conceptual sophistication ***	2.28	3.67
Join change unknown conceptual sophistication ***	1.78	4.19
Multiplication conceptual sophistication ***	1.76	2.86
Part part whole conceptual sophistication **	1.46	2.55

NB: T-test used ***p<.001; **p<.01, *p<.05

The analysis identified significant increase in conceptual sophistication of Primary 1 and 2 pupils from pre-test to post-test.

In order to examine the potential effect of CGI intervention, we compared the Post-Test outcomes of P1 students against Pre-test outcomes for P2 children. This is because we would expect P1 students at post-test to be at a similar stage as P2 students at pre-test. The analysis there enabled us to determine if there was a greater proportion of P1 pupils judged as secure in the mathematical domains after CGI than would have been expected (i.e., proportion of P2's who were secure at pre-test). In other words, are P1 students now doing significantly better than where P2 students were without CGI?

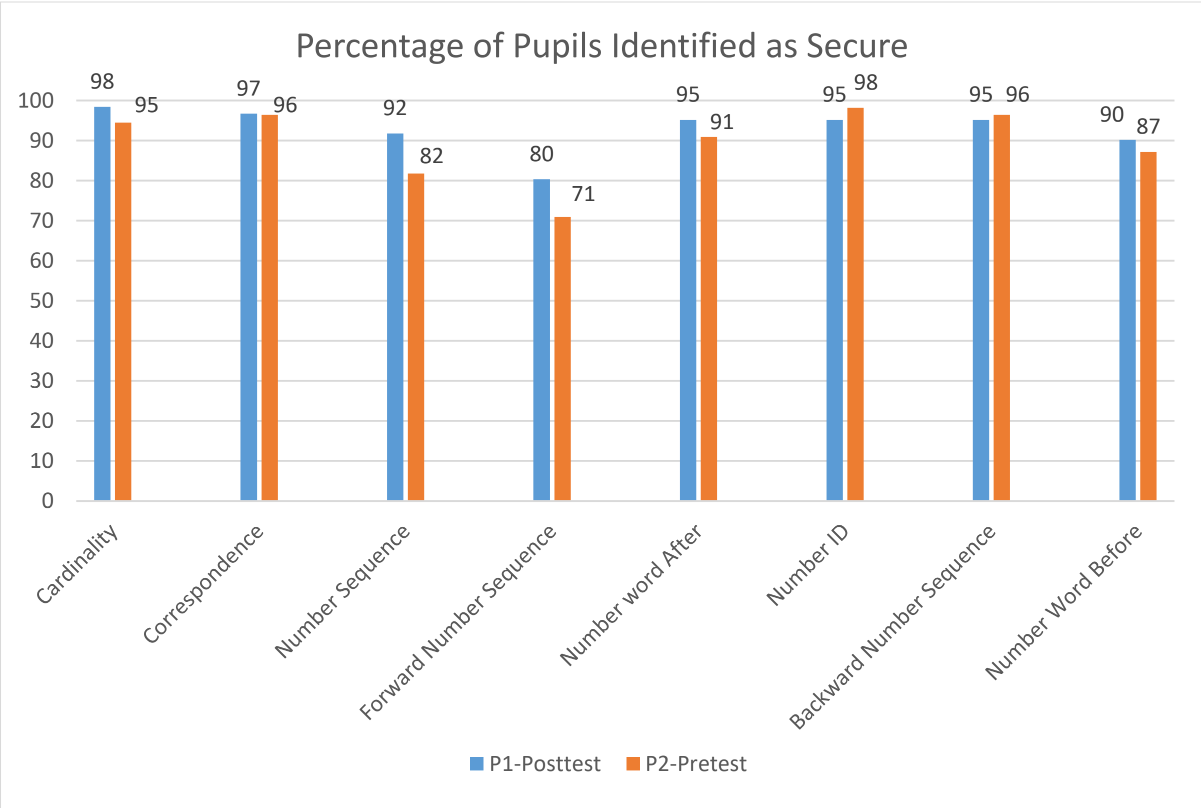


Figure 8: Difference in Post-Primary 1 and Pre-Primary 2 Results on a range of mathematical domains

The trend from Figure 8 above indicates that more P1 students who received CGI intervention at post-test were judged as secure on at least 5 of the 8 domains measured. A statistical test (using Fisher's Exact test) suggests that these differences were not significant. However, considering that this intervention took place over a school year, and that P1 students were younger at post-test, than P2 at pre-test, it can be argued from the trends that CGI intervention shows promise in helping a larger proportion of children's improve their mathematical ability. Future interventions that look at trends in performance of pupils not taking part in CGI as well as attainment of different categories of pupils will be useful.

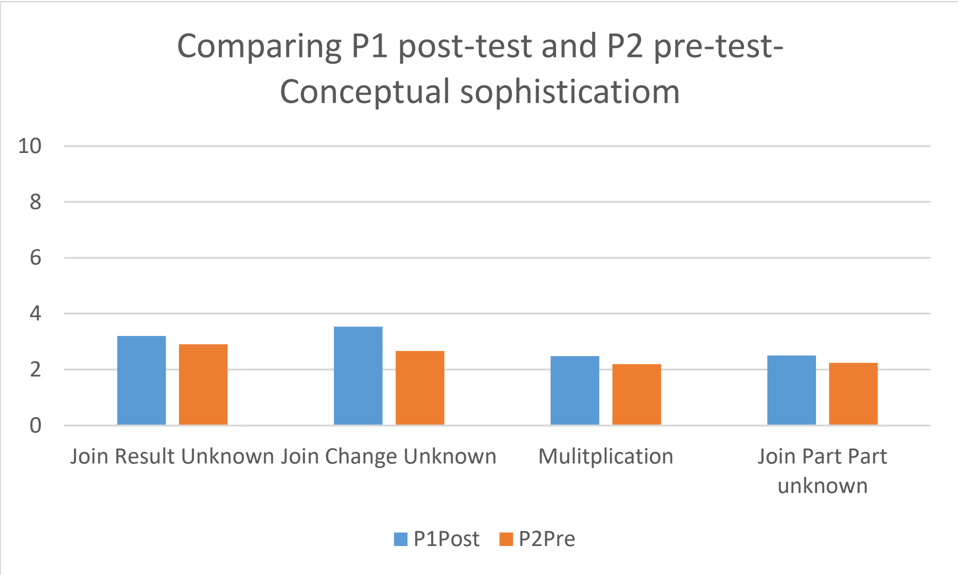


Figure 9: Comparing Primary 1 Post-test and Primary 2 Pre-test Conceptual Sophistication

Trends for conceptual sophistication also suggest a tendency for higher scores for P1 students at post-test in comparison to P2 pre-test scores. However, these differences were not statistically significant. As in the case for analysis on proportion of students who were secure, it can be argued that CGI shows promise but requires further examination.

Exploring Links between Classroom Experience and Attainment in Primary 1

Evidence from the quantitative statistical analysis of the assessments presented above shows an increase in performance in most mathematical domains and in the children's conceptual understanding. The following excerpts from classroom observations, interviews and fieldnotes provide an insight into some of the underlying processes which contributed to these outcomes. These data are situated in the real-life context and reality of the classroom and provide an account of the interactive nature of teacher moves and instructional decision-making along with the classroom experiences of children. They also demonstrate changes in teacher knowledge and beliefs to which we will return later.

Early years' teachers in particular commented on the positive impact of a greater emphasis on counting within the classroom.

"I've found that with using the CGI and other oral counting strategies, the kids are much further forward than any primary one class I've had before. …

…we've got some of the primary ones now who can count way beyond thirty, which is what they're expected to do for the end of the year. Some of them are now at the stage of working out how to write one hundred and one, and you know, they're really pushing the boundaries."

Primary 1 Teacher with 18 years' experience

Figure 10: Primary 1 observation, three pupils demonstrate making 20

"I think they understand better the relationship between numbers as well. They're much quicker with their number bonds to ten, and it's not necessarily- they can rhyme it off the top of their heads.

..now they're counting in twos, and threes, and fours and fives, whereas before they wouldn't have been encouraged to count in threes and fours….

Primary twos are counting into their hundreds. Primary ones, most of them can go into the fifties, sixties, seventies, which is really good in comparison to where they would've been last year, or if we hadn't been doing the CGI."

Primary 1-2 Teacher with 2 years' experience

Teachers told us that they were feeling confident enough to introduce concepts much sooner to early years pupils.

> “In particular we’ve also been looking at multiplication and division which is something we would never have done in primary one before

The other thing too, I’m beginning to see place value coming through which is something that Cheryl helped with. So that the children could see when they counted to ten and put it into a bag, that was ten and four ones would be fourteen. Ten, and four ones, equalled fourteen. So we’ve got the understanding of place value as early as primary one coming in.”

Primary 1 Teacher with 36 years’ experience

Primary 1 Strategies

The problem: Cook made 12 slices of pizza. There are three people having dinner.

How many slices did each person get?

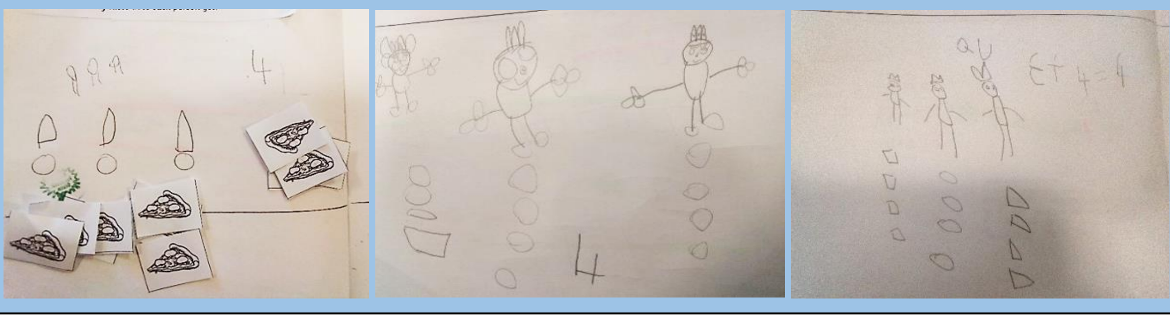


Figure 11: Extract from Primary 1 Observation

Teachers told us that children were more engaged in mathematical activity.

“children were all talking to each other and walking round the classroom – not in an aimless way – every single one of my children were completely involved in the task and really enjoying it. You’ll see children using different strategies, using different materials, working things out. It’s just a really lovely environment….

before when we did the Heinemann maths scheme, they would be sitting with a book. I’ve got quite a few children in my class that completely zone out. Behaviour starts to get worse because they just don’t enjoy doing it. And I don’t have that problem anymore. They are so independent! Like, I would say, here’s the problem. And before I’ve even finished saying the problem, they’ve already got up to try to find things to solve the problem because they’re so interested.”

ASN teacher with 8 years’ experience

Primary 3 Attainment

The Statistical Evidence: difference in Pre-test and Post-test mathematical conceptual sophistication and solution strategy of children

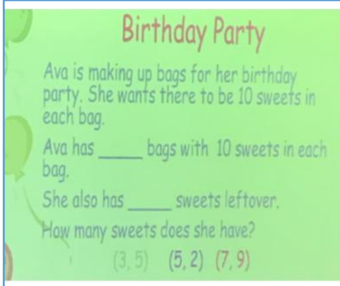
We used 2 questions to examine if there was a change in the mathematics problem solving abilities for P3 children across the 3 schools. Solution Strategy and conceptual sophistication of strategy were measured as outlined above, with conceptual sophistication on a scale of 0 to 10.

Table 17: Primary 3 - difference in Pre-test and Post-test mathematical conceptual sophistication

	School 1 to 3 (n=60)	
	P3 Pre-test average	P3 Post-test average
Join Change Unknown conceptual sophistication	4.13	4.90
Multiplication conceptual sophistication	2.68	4.45***

Results for T-test (Table 17) indicates significant improvement in the solution strategy and conceptual sophistication of pupil's strategy in solving of multiplication problems. Even though post-test scores were relatively higher for Join Change Unknown Problems, these differences were not statistically significant. Caution is required in making attributions about the impact of CGI as we cannot tell if pupils performed better than would have been expected due to an absence of a control group. Equivalent P4 pre-test data was also not available for comparative analysis. However, the improved outcomes at post-test indicate a trend in the right direction.

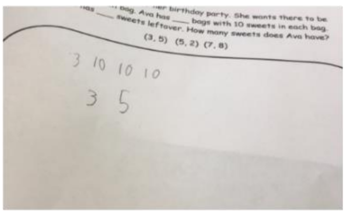
Exploring Links between Classroom Experience and Attainment in Primary 3



Extract from Research Team Observation Notes

Pupil direct modelling 1 bag of 10 but then went to remove 5. She didn't realise she needed to work out all 3 bags before working with the additional set.

She then modelled out all three bags and additional 5. She counted from one but was then also able to count the bags in tens.



"Three tens make 30 and add 5 on makes 35"

Could this number sentence be developed to record with symbols..

Sophie writing 10 in each bag and then counting in tens. Good questioning in sharing from Cheryl and Louise pushing children

Figure 12: Extract from Classroom Observation

Teachers in the lower to middle stages of the primary schools were consistently positive about the impact of introducing CGI. Teachers remarked that children were exceeding their expectations, they were further on and tackling more challenging mathematics than would typically have been expected of them, or indeed planned.

"I think it's absolutely the way I want to keep on teaching, because having seen the amount that the children get out of it, and how much further on I've managed to get them at primary three, more further on that I'd ever have got them with a resource-based scheme and the sort of, older way of teaching. ...

With the fractions work we were doing, we were able to move well beyond finding a half, a quarter. Groups of my children were going on to find equivalent fractions, beginning to simplify fractions, working with tenth, eighth. They seem to have so much of a better understanding. The same with equations. Having an understanding of equations, and how they have to balance an equation, I would never have gone to that kind of work before. "

Primary 3 teacher with 13 years' experience

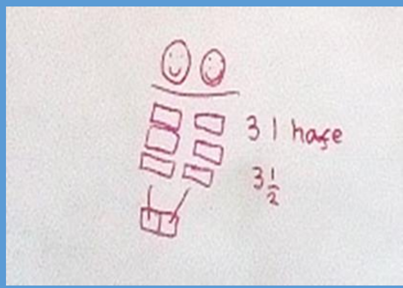
Teachers were beginning to look beyond children's ability to carry out procedures accurately with an increasing focus on their conceptual understanding.

"For those in the sort of middle group, as I said before, there's a couple of children, it's highlighted the gaps that we've known they were quite confident with maths, but they might have been doing formal method sums, but they might not actually fully understand why is that twenty, not a two. "I've written four here, but you're saying that's four hundred", and they didn't actually know why it was."

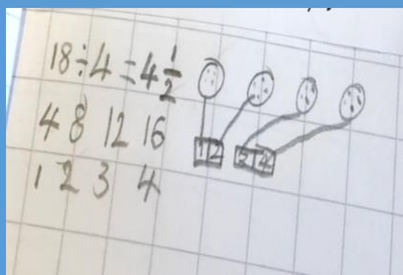
Primary 3-4 teacher with 7 years' experience

PUPIL STRATEGIES

The teacher has 7 Mars Bars to share between 2 friends so that everyone gets the same. How many Mars Bars will each person get?



The teacher has 18 Mars Bars to share between 4 friends so that everyone gets the same. How many Mars Bars will each person get?



"I counted in fours to 16 . I knew I couldn't add on another 4

so I halved both of the other pieces and I knew everyone

could get 1 half."

Primary 3 Pupils

Teachers also commented on that many children who had previously struggled were growing in confidence and developing an ability to explain their thinking:

> *For some of them in particular, there's two of them who could not do a single problem, they were really struggling, they were really upset, now they're doing really well, not soaring ahead, but their confidence is much better, their understanding is much better and they can explain what they're doing really, really well.*
> Primary 3-4 teacher with 7 years' experience

Primary 4-7 Attainment

The Statistical Evidence: difference in Pre-test and Post-test mathematical conceptual sophistication and solution strategy of children

Table 18: Primary 4 to 7 - difference in Pre-test and Post-test mathematical conceptual sophistication

	School 1 to 3 (n=157)	
	P4-7 Pre-test average	P4-7 Post-test average
Division Fraction conceptual sophistication	3.04	2.87
Open number sequence conceptual sophistication	2.87	4.20 ***

We used 2 questions to examine if there was a change in the mathematics problem solving abilities for children across the 3 schools. Solution Strategy and conceptual sophistication were measured in the same way as described above. Overall, results for T-test (Table 18) shows a significant improvement in the sophistication of strategies and conceptualisation in solving open number sequence problems but not for division fraction problems among P4-P7 students across the three schools. Unlike the case for P1 and P2, it was not possible to test whether pupils had higher solution strategy and conceptual sophistication of strategy at post-test as questions of different difficulty levels were used for the pre-test at higher classes.

Table 19: Primary 5 & 6 pupils - difference in Pre-test and Post-test conceptual sophistication

	School 1-3 (n=97)	
	P5&6 Pre-test average	P5&6 Post-test average
Division Base 10 conceptual sophistication	4.19	45.14**

Results for T-test (Table 19) shows a significant improvement in the sophistication of strategies and conceptual sophistication of strategies used in solving division base 10 problems. However, we cannot tell if this is different from where the pupils would have been without CGI.

Table 20: Primary 7 Pupils: Is there a difference in Pre-test and Post-test mathematical conceptual sophistication?

	School 1 & 2 (n=19)	
	Pre–test (P7) average	Post–test (P7) average
Join Change Unknown conceptual sophistication	4.67	7.21***
Multiplication conceptual sophistication	5.54	7.15***
Division fraction conceptual sophistication	3.03	3.90
Open number sequence conceptual sophistication	2.74	2.62
Division base10 conceptual sophistication	4.38	6.41**

Both schools 1 and 2 provided data on P7 students. However, the data were obtained at different transition points. While School 1 took Pre and Post measures in P7; School 2 had Pre-test at P6 and Post-Test at P7. The combined data, using T-test, suggest statistically significant increase in test scores for strategy and conceptual sophistication for Join Change Unknown; Multiplication Strategy; and Division base10 problems. However, as in the other analysis it is not possible to determine how much of this change is attributable to participation in CGI.

Exploring Links between Classroom Experience and Attainment in Primary 4 to 7

In the upper primary, there was more of a challenge in introducing CGI to children whose prior experiences would have been explicit demonstration of strategies. However, over time, there was evidence that children's reliance on teachers' demonstration dissipated, children were making sense of the problems through actively engaging with the structure of the problems. This included children who had previously struggled, for example in relation to a child with a specific learning difficulty:

"I've noticed in the upper school especially, is that children are becoming more confident with the worded questions, and breaking them down, and they're pulling the numbers, and they're not just saying like, "groups of…", and automatically thinking multiplication, they're going, "oh, is it multiplication or is it something else?", and they're really thinking about that. Particularly one wee boy I was working with, he's actually got dyslexia, and when faced with worded questions in the past, it's really put him off. Now his maths, I was working with him, he was doing an assessment for us. It's the government assessment, and his maths results at the start of the year were quite poor. Then working with him recently when I was working with the worded questions, having them read out to him to get over his dyslexia, but actually, he was breaking the numbers down, he was seeing the links between numbers and he was, he scored quite well. So just seeing the difference with that one child who's been working in Anna's class, and then been seeing if they take like, three hundred and ten, that really they're breaking it down into tens, hundreds, what does that mean? I could see that when he was working, and it's really increased his confidence. So it was really great to see a child who had a barrier because of dyslexia, for worded questions, then actually he was working with it, and he was enthused. And I think had he maybe just been faced with a calculation that wasn't in a worded problem, maybe, I don't know if he'd have the same enthusiasm. I think it makes a big difference."

Primary 7 Teacher with 5 years' experience

The reliance on teacher demonstration was particularly evident in the upper primary. Teachers were also concerned about how children might cope with the transition to secondary school and felt that they needed to be equipped to deal with the expectations of high school, whatever these might be. This created a tension between supporting children in developing conceptual understanding through their own sense-making and ensuring that they developed the computational fluency required for secondary. This issue featured strongly in the Saturday workshops and school-based sessions. It became clear to the teachers that this was a false dichotomy and that pupils' sense-making and computational fluency should note be seen as separate. Teachers recognised the importance of allowing children to use their own sense-making strategies and were reassured through collaboration across schools and discussions led by school-based co-ordinators which helped them to connect these strategies to more traditional algorithms. Central to this was recognition of the importance of engaging children in mathematical sense-making in the first instance.

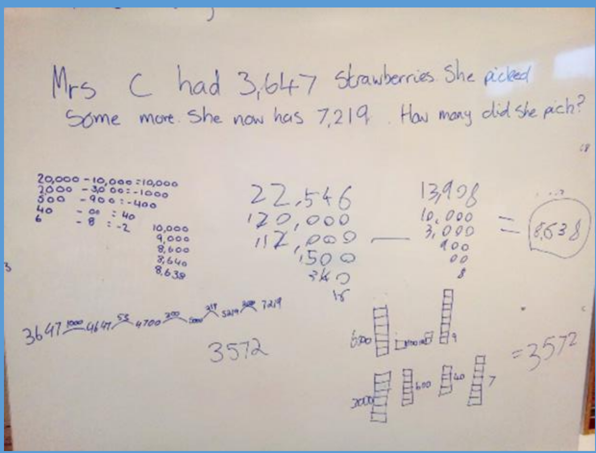
"I just feel that CGI has made a big difference. Even for the kids at the start of the year I feel the kids were reliant on the teacher telling what to do and I've seen a big improvement on that. They're more willing to work things out. They're doing things I don't think they would previously have covered at that stage

…/…

they're more likely to not give up on a problem, before if I've given them a multiplication say 3 digit by 2 digits, they would go 'no, cannae do it'. But now they've got different ways of writing it down, working it out, I've seen a big difference."

Primary 7 teacher, with 8 years' experience

PUPIL STRATEGIES



Primary 7 Pupils

Figure 13: Extract from Primary 7 Observation

Changes in Teacher Knowledge and Beliefs

This project's theory of change rests on changes in teacher moves based on sharing three areas of knowledge: mathematics, teaching approaches; and knowledge of learners. Chapter 5 set out the interventions used to share this knowledge, and this section sets out findings with respect to changes in teachers' knowledge and beliefs and highlights how closely the three areas of knowledge are related. The next section goes on to reflect on changes in elements of instruction used by teachers following such changes.

Qualitative data from before and after interviews with teachers has been used to highlight those areas in which teachers acknowledged change had taken place.

Subject knowledge

Initial classroom observations and professional dialogue revealed that subject knowledge was an (often unacknowledged) issue for some teachers. A number of teachers appeared uncertain about commutative, associative and distributive principles, different problem types, such as distinguishing between partitive and measurement division, and particularly the potential implications for learners.

It might not make a difference if they had a problem like … "how many sixes are in twelve", or "do you have to put it into six groups of two or two groups of six", because you know that that's the same outcome. It's two different concepts. I can understand why when I spoke to the teacher this morning she felt, "this is what I'm not sure of", and "these are the things I'm a bit concerned about. Do I need to make sure that they understand that these are different concepts?"

Headteacher

Figure 14: Extract from Primary 1 Observation

While the project did not seek to focus directly on teachers' own knowledge and understanding of mathematics, teachers were introduced to new ways of looking at problems, how pupils might tackle them and why some problems are more challenging than others, illustrating how closely intertwined are the subject and pedagogical knowledge. By concentrating on promoting knowledge about how children understand mathematics, teachers also boosted their own subject knowledge and confidence in teaching mathematics.

"I think it's really helped me, and my own understanding of maths, just general maths. My mental maths has certainly got better through it, and I feel that I'm able to explain better, the way that the number process, all that kind of thing works. Even things like place value and multiplication, all those things, my class aren't really at the stage of complicated, decimals, and fractions and all that kind of stuff, but the basic stuff, I feel much more confident in being able to teach, and being able to explain it."

ASN Teacher, with 4 years' experience

"I just feel like I've got a much better understanding in terms of division problems, multiplication problems, addition, subtraction I feel like my understanding of all that is much better, which I feel gives me more confidence when I'm teaching"
Primary 1-2 Teacher, with 2 years' experience

"I got my standard grade and my higher but it was a chore, whereas now I'm going back, "oh! That's how that links to that!""
Primary 1 Teacher, with 18 years experience

Jacobs et al (2010) note the close relationship between the skills required in professional noticing, specifically the need to identify what is mathematically significant in attending to children's strategies and the mathematical and pedagogical knowledge required to respond appropriately.

Pedagogical Content Knowledge

This was a core element of the conceptual framework of the project and is described more fully in Chapter 3. During the project teachers were encouraged to think beyond an over-simplified approach to teaching informed by two primary components: subject knowledge and pedagogical knowledge. Instead they were encouraged to reflect on how various aspects of knowledge for teaching combined to inform practice and in particular how knowledge of children's mathematical thinking was part of this mix. Mathematics was an area of teaching that some teachers lacked confidence in and some felt ill-prepared through their initial teacher training for teaching numeracy.

"I don't know that I'm confident in teaching maths completely. I think at uni I was never properly taught how to teach maths, and I think it was just a case of, "these are your resources and…", whereas with literacy you have everything all step by step, but with maths I don't have that. So I don't feel as confident teaching numeracy as what I do teaching literacy."
Primary 1-2 teacher, with 2 years' experience

"I did a four year B.Ed. and didn't really know much about how children develop their maths understanding. I'd no idea that you could do your multiplication and division with using the concrete materials and drawing out diagrams and stuff"
Primary 1 teacher, with 18 years' experience

Teachers were keen to learn more about children's mathematical thinking and development. Several teachers remarked that they had previously found themselves not understanding how a pupil had arrived at an answer, or feeling that they needed to offer their pupils a wider range of approaches or strategies to help them. New insights into children's mathematical thinking, focused teachers' attention on how they might develop activities and problems to develop pupils' understanding and individual strategies rather than earlier emphasis on repeating taught algorithms.

"I think the way that we've been shown, in Heinemann maths it's teach them a strategy, and that's what they're going to do…. From what I understand of CGI it's a completely different approach, and it's more freeing the children up isn't it? To sort of let them come up with ideas? The way I see it, is to try and arm them with different strategies if you can, and they can pick what suits. But if they come up with something different and it works, then I'm happy with that. (Initial Interview)

I think at the start I just thought it was problem solving and that's it, and that you don't actually do any teaching. But from observations and seeing Lio and Cheryl, it made me realise, well, grab those teachable moments, and that's your time to go, "Well, what you, Child-A actually did there, was this", which is good because it's a bit more organic, rather than standing at the board and saying, "this is the way". "

Primary 3-4 teacher, with 7 years' experience

"reading the [CGI] book that all teachers were given and liaising with some colleagues and reading their notes from doing the module, I've learned a wee bit about the stages that children will come to, starting with the direct modelling, using derived facts, known number facts. And very much being led by them and the way they approach problems."

Primary 3 teacher, with 13 years' experience

The project gave teachers new knowledge about the foundations of children's mathematical thinking as well as approaches for teaching and assessment that they were able to work on in the classroom. Skills such as problem setting, professional noticing and recording all helped teachers to better assess their pupils understanding and give them a base from which to develop.

"The session that we had with [Professor] McLennan … that affected me a lot because I realised that I hadn't been doing enough of that

…/..

the way she described the stages of counting, that had never been described to me before

…./…

made sure that there was all those counting activities, and that we built them into the routine, and I think that definitely benefitted the children. And it let me observe where there were gaps in the children's learning that you might not have picked up, just in the normal course of things. But putting all those counting activities on and really focussing on it let you see that yeah, there were gaps here, and what you need to focus in on to try and sort that out before moving the children on. "

Primary 2 teacher, 14 years' experience

"I feel before there might be, you know, a child might do something, and I'd think, "I've no idea, why are they doing that?", whereas I can go back and say, right, they've not got their base ten, they don't know-, they can't work out-, so their place value is off, or even their number sequencing, you know counting up in their threes, isn't there so skip-counting, that needs more work, whereas these were things that, I don't know, that I thought children would automatically know. Whereas now I can see where things need to be solidified before the children can go on. Looking at the way they count up, so a lot of them will still count up in ones, but then you can see the advancement of the one's who've got, that they know their times tables, they understand their times tables are much quicker going up in eights, you know, different things like that, so that's made a big difference in how I teach."

Primary 4/5 teacher, 3 years' experience

"I think [teachers] need to know ….all the different stage of development and to link that up with the problem types, and the number combinations that we're using. I think you do need to have an understanding of that, before you go onto the problem solving context. So what are the different problems asking the children to do? What strategies could come from the problems that you set? And I think if you've got a knowledge of that developmental process of the stages of development of the child, you can then use that to construct a problem so that you're moving their development on."

Primary 2 teacher, 14 years' experience

Teachers identified children's intuitive knowledge and development of mathematical thinking as new knowledge that made a difference to them. These ideas were initially expressed in broad terms but became more specific as the project progressed.

Some teachers expressed more detailed and specific knowledge relating to understanding what happens in the mathematics classroom - what Jacobs et al. (2010) refer to as 'professional noticing'. These included references for example to 'strategies', 'problem types', and of 'choosing appropriate numbers to allow the children to use different strategies'. Jacobs, Lamb, & Philipp, (2010 p.196) draw attention to the movement from the general to the more specific as indicating growth in the context of professional noticing.

Responses generally indicate that participants understand CGI as fundamentally different from 'traditional' practice in which teachers need to explicitly demonstrate procedures and children learn from repeated practice. This suggests that most, if not all, participants recognised 'traditional' practice as what Carpenter et al. (2000) refer to as Level 1 – "*teachers believe that children need to be explicitly taught how to do mathematics…*"

There are indications that some teachers have reached Carpenter et al's Level 2 - where teachers "*begin to question whether children need explicit instruction in order to solve problems and the teachers alternatively provide opportunities for children to solve problems using their own strategies and show the children specific methods*".

Some teachers had opened up their thinking to new ways of working particularly around spending more time in dialogue with children about their strategies and understanding.

"I think it's about being flexible and giving the children the opportunity to do their own thing and come up with their own strategies. The importance of the teacher is really the questioning and asking children different types of question to deepen their understanding. To make them more confident at explaining as well."
Primary 7 teacher with 8 years' experience

"I think it's about working form where the children are at, and developing their understanding of the number patterns, of the number relationships, rather than teaching a process that they then just follow whether they understand it or not."
Primary 1 teacher, with 18 years' experience

"before I would've been so concerned with them doing it the way I was telling them to do it, whereas now I feel like it's kind of ok to just to be, "show me how you're going to do it. You make sense of it". "
Primary 1-2 teacher, 2 years' experience

Responses also reveal some uncertainty about the supporting pedagogy required. Several teachers point to differences in structuring learning for example, and some present this as a movement from 'traditional, structured' learning, to 'informal', more discovery-based learning. In the early stages some teachers struggled with what they perceived as a lack of structure and looked for a single common development route for pupils – a step by step guide to learning through CGI. Over time, most teachers developed and used a new 'language' for communicating pupils' mathematical development, and demonstrated more confident professional judgement about how and when to move classes and individual pupils on.

In general participants' responses indicated that their understanding of 'knowledge' of children's mathematical thinking is inseparable from the skills required to teach.

Changes in Teacher Moves

This section identifies the key changes in teaching practice that were observed in classrooms over the course of the project, including:

- Increased use of counting and related activities in the lower school
- Increased use of word problems and focus on mathematics
- Pupils encouraged to find their own solution strategies
- Increased dialogue and discussion about mathematics
- Changes in assessing and recording pupils' understanding

Counting Activities

Teachers reported increasing awareness that they had been making assumptions in the past about pupils' number recognition, understanding of numerals and their relationships. Teachers showed increasing awareness of that in their teaching.

The use of word problems

Teachers were quick to associate the use of word problems with CGI and to introduce problems they constructed for themselves to their classrooms. Previously this way of framing problems had only been associated with 'problem solving' activities and had largely come from text books and worksheets. Gradually we saw teachers reducing the number of problems set for pupils and an increasing number of teachers by the end of the project could spend an entire lesson working on a single problem with their class. This was in stark contrast to setting a large quantity of number based problems to be solved with a single algorithm.

> **I was worried myself, at the start of the year, working on one problem for the whole session, and I was thinking 'oh my goodness' because usually we're giving them fifty sums to do you know, in a session, and I'm thinking, 'one sum?!'. But, actually, the maths that we're pulling out of that one thing, and the different ways of solving it has been incredible really.**
>
> Primary 7 Teacher, 5 years' experience

By focusing on a very small number of problems, teachers were able to engage pupils with a wide range of abilities using different number combinations for the same problem type, so lessons became more inclusive.

Some teachers found it easier than others to devise effective word problems, but were assisted by the collaborative planning of the Lesson Study approach which encouraged teachers to consider the interactions between the mathematical concept they wanted pupils to explore, the phrasing of the problem, the number choice and how pupils responded.

Supporting pupils to find their own solution strategies

Move toward encouraging pupils to find their own (and/or multiple) solution strategies and away from demonstrating procedures. Moves to support this included:

- encouraging pupils to explore problems in their own way and in particular, through the use of materials;
- asking questions of pupils about their thinking rather than demonstrating solution strategies;
- spending time ensuring pupils understood the problem and what was being asked of them;

- encouraging an exploratory and collaborative ethos in the classroom in which unsuccessful strategies could be learned from and away from sole emphasis on 'the right answer'.

Early in the project some teachers found it difficult to move away from demonstrating one or more solution strategy, but observations and interviews confirm that teachers increasingly sought to introduce choice, particularly in terms of materials. Latterly, teachers were encouraging pupils to use their prior knowledge and identify their own solution strategies.

> **before I would've just sat down, "well this is how I've shown you, do that.", whereas now I'm sitting back much more or asking them questions about what they already know or, "how did you get to that answer?"**
>
> Primary 1 Teacher, 18 years' experience

Pupils of all ages were encouraged to make use of materials and drawing to represent their thinking. Teachers reported considerable resistance among pupils in the upper levels (P5-7) to the use of materials, although some teachers appeared to associate the use of materials so closely to CGI practice that pupils were expected to use them even when unnecessary. Many pupils in the upper levels (P4 up) remained committed to formal taught algorithms. This is consistent with previous studies and experiences across school in the Scotland and in the US. Pupils show a reluctance to engage in problem solving as an authentic process of investigation preferring instead to work within the security and arguably less challenging confines of carrying out a taught procedure regardless of whether it is actually understood.

Mathematical Discussion

Mathematical discussions in the classroom increased in frequency and with most teachers became a routine element of practice.

> *"when I'm working with the children, when I'm moving around the class, there's a lot of questioning. "Is there a quicker way you could have done that", or, maybe not so much quicker, just trying to move them on in their understanding. If I know that they can skip count, just trying to encourage them, "well did you need to count in ones? Is there something that we know how to do that could help us here?" Also recording their thinking formally to show them the link between the written algorithms and also doing that at the end of a lesson when you're doing your plenary that, that's actually not really at the end of the lesson, it's a major part of the lesson, probably about more than a good third of your lesson. Asking the children to demonstrate to each other, and again linking their thinking to the formal recording."*
>
> Primary 3 Teacher, 13 years' experience

Primary 1

using Flipgrid to record and share their accounts of how they solved the problem

Figure 15: Extract from Primary 1 Observation

Teachers began to demonstrate a more strategic approach to asking pupils to present their strategies to the class, enabling some students to set up the problem, while others would demonstrate strategies of increasing conceptual sophistication and draw out connections and concepts. Lesson study supported this too, as teachers had already anticipated the types of strategies they might see and to think about where to take the mathematics.

> *"it is a big challenge! It really is. But they love it, they love getting up in front of the class and saying how they managed to- I think because there's an ethos set, of… I mean there is a right answer, but there's not a right approach, and I think the children relax into that. So they don't automatically think, 'am I getting this right?', or, 'am I getting this wrong?' They know that, yes, there's an answer that they need to find. There's a problem that they need to solve. But, that they have the freedom, and exploration to try different things and find a way that works for them. So, I think the pressure's off in terms of, you know, getting it right or getting it wrong, and I think that does set a good ethos in the classroom."*
> Primary teacher with 7 years' experience

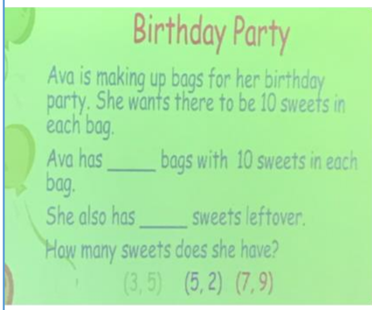
Teachers participating in lesson study were operating at Level 3 on Fennema et al (1996), scale. Teachers did not present procedures for pupils to imitate, anticipating they can solve problems independently. Teachers applied their understanding of children's thinking to their choice of problem and at times to the choice of topic. Children spent most of the class solving a problem and reporting/comparing their strategies. Some teachers were able to recognise and respond to particular challenges facing individual pupils and derive problems/questioning that supported their development. None of the teachers evidenced Level 4 in their practice, which is to be expected given the relatively short time most of the teachers had engaged with CGI.

Assessing and Recording Pupil Strategies

> *"I think previously it would be just your summative assessment at the end of a topic. I think now you have to really be able to give a child a problem and be with them to get an understanding. Are they counting from one, are they able to count on from different numbers. Do they need the concrete materials? Or do they have any known number facts already. I think it's quite complicated. It's a lot of observing and various ranges of problems to gain an understanding."*
>
> Primary 3 teacher with 13 years' experience

Teachers put considerable effort into building their professional noticing skills and as a result found that they accumulated a great deal of knowledge about the mathematical understanding of individual pupils. Teachers devised various ways to annotate pupil jotters and to maintain an overview of progress across the class.

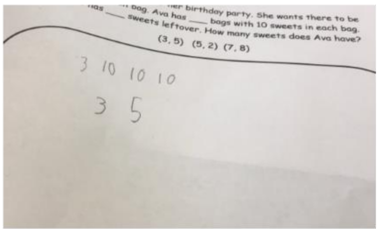
An ongoing concern among teachers in the upper primary classes was how best to track progress through the curriculum and one of the CGI co-ordinators led the way in deriving a template to assist teachers to make these links.



Extract from Research Team Observation Notes

Pupil direct modelling 1 bag of 10 but then went to remove 5. She didn't realise she needed to work out all 3 bags before working with the additional set.

She then modelled out all three bags and additional 5. She counted from one but was then also able to count the bags in tens.



"Three tens make 30 and add 5 on makes 35"

Could this number sentence be developed to record with symbols..

Sophie writing 10 in each bag and then counting in tens. Good questioning in sharing from Cheryl and Louise pushing children

Figure 16: Extract from Classroom Observation

Changes in Inclusion

In spite of the fact that the development of inclusive practice was explicit in our aims and research questions, during the various professional development sessions and activities throughout the project with the teachers **we never spoke explicitly about inclusion. We focussed on pedagogy** and we watched as teachers developed their practice which came to be transformed in a more inclusive way. At the heart of our work with the teachers was the principle of supporting the development in practice of a process of dynamic assessment which illuminates learners' needs on the basis of actual current knowledge and understanding, rather than on the basis of identifying any gap between where a child is, or should be, within a particular curricular framework.

There were two striking findings relating to practice in the schools in this respect. One was that all three headteachers stated at the end of the project that they would no longer continue with ability grouping in their schools as it was felt that these groupings did not necessarily reflect the abilities and understandings of the children within the groups. Perhaps even more powerful evidence was presented by the teachers across all three schools, who stated that over the duration of the project not a single child was deemed to require additional support for numeracy outwith the classroom. This does not mean that no child needed support with numeracy, but rather, where individual children were identified as requiring support this was dealt with through restructuring classroom practice.

"It has totally, in terms of the grouping that we have previously had, it has really thrown it up in the air because it is completely inclusive now, completely inclusive. Now children that we would have regarded as high fliers are working great and doing really well but they are being exposed to mathematical problems and word problems so they are able to apply their knowledge. The biggest impact has been on the children who we would have regarded as not being the best at Maths. They are doing amazingly well because it makes sense to them."

Headteacher

Having more knowledgeable teachers means that they are more confident in responding to the needs of individual children and the evidence in our project was that they were able to do this in the context of the classroom.

"I think having the confidence to not follow a textbook but work with the children… I know now, more where my children were than I have in previous years. I would've collated them as a group and said, oh that group can do X, Y, and Z. But now I can see more individually their mathematical understanding."

Primary 2 Teacher

This is very different to a remedial model of support which involves the removal of a child from the classroom to work with a teacher, or student or pupil support assistant, whose knowledge of children's mathematics may be limited, before returning them to the class in which they were struggling. In our project, we saw a transformation of practice in the classroom which supported the learning of all children in those classrooms in ways which helped prevent failure in the first place. This is not to say that some particular children may not on occasion benefit from intensive one to one intervention but this support needs to be focussed and specific. In the project where this was the case, it happened within the context of the classroom. All three headteachers described how support practice in their schools had changed.

> *"No we have no children coming out for any different interventions, everything is done through CGI …it is certainly easier from the point of view of staffing because we don't have a massive resource of staffing. So it has certainly made that a bit more effective. Also the class teacher is more responsible because in the past, children being extracted to do catch up on numeracy or whatever else, that kind of takes the responsibility of the class teacher as well and thy just put down into their plan working with Mrs so and so catching up on certain concepts. Now that Maths is much more inclusive, the responsibility very much lies with the class teacher."*
>
> Headteacher

> *"I think there are more children coming through and being able to achieve and excel who would previously have been fixed on a path (in ability groups) but now we don't. Don't get me wrong we still have an idea within the classes, you've got to have who is your more able etc. and what are we doing to impact on that? We still have a few children who are on intensive support but they still, in terms of the teaching of numeracy, they are included in what the rest of the class are doing."*
>
> Headteacher

> *"What I found from my staff is they will very quickly be telling me little anecdotes about children who are in the less independent groups of Maths, about how much their confidence has come on, I've heard about how much their attainment has progressed and actually, staff themselves are now able to think about the children's learning in a much, much deeper understanding and knowledge base than they had before. I suppose in essence really the children's attainment has come on and equally so has the staff. It is not just about the children."*
>
> Headteacher

This change in practice was reflected in teachers' comments, they also recognised the benefit of more inclusive practice.

"None of the kids go out in primary one. Out of fifty children, not one single child goes out… Yeah, it's really nice for me because, well, my concern before was with groups going out, they were always missing out on something else in the classroom… I think they (benefit) academically and socially. I think socially they stay in and it's a confidence thing, they're not having to go and work in a little group. I've tried like the flexible grouping, so that it's not the same group that they work with every time, but every single child is included in every single lesson. Then, mathematical wise, I can have challenges ready for some of my children, but I also know, who maybe couldn't cope with the whole problem, but could have a special job even just counting out the first part of the story."

P1 teacher

Schools adopted a flexible approach to responding to individual children and support came to be more focussed and structured within the classrooms.

"Because we work now in mixed ability groups and because the groups are much more fluid, you basically don't have children being extracted to work in numeracy on a one to one with a support assistant, that doesn't happen now. It is very much embedded in what they are doing in the class and I think it allows the staff to differentiate in terms of the problems and what they are expecting from the children and what strategies they are going to use. I think personally, seeing it in action, it is much more subtle than we used to have children who were on Heinemann 2 and were in Primary 4 and you would have other children in Primary 4 who would say I'm on this book they are only on that book. With CGI it is much more subtle."

Headteacher

One headteacher cited the example of a pupil in an ASN class who,

"would've have been probably one of our less able and he is now working in the mainstream class Primary 5 top group for numeracy and that happened within a matter of a year.

"So in terms of what we know about children and being able to include and shift children around, it's been instrumental in driving that agenda and helping with the inclusion in terms of the children moving across classrooms."

Prior teaching was seen as a source of learning difficulties.

"I've found one girl in my lower ability group is flourishing with this approach and her confidence has come on so much. Previously I would have said she's somebody I would have had to give quite a lot of additional support to. But it was obviously just the approach that was being used that wasn't effective for her learning. So, as I mentioned earlier, also the girl in my top group who's working at second level division, previously might have been curtailed to first level and moving on next year. But it's how to manage that. Because I'm finding that within groups there are groups. It's just a challenge to manage this as a class.
Primary 4 teacher with 13 years' experience

The teacher of one ASN class described the engagement of children in mathematical activity in ways that supported independence and resulted in less challenging behaviour.

"Children choosing themselves how they're going to tackle problems. Oh, and first you'll see children tackling taught problems rather than sitting at workbooks. A teacher came in to my class today and children were all talking to each other and walking round the classroom – not in an aimless way – every single one of my children were completely involved in the task and really enjoying it. You'll see children using different strategies, using different materials, working things out. It's just a really lovely environment. It's really, really nice. You'll see…. independence ... I know its maths, but independence. I don't know if it's just being in the ASN department, but I'm just amazed because before when we did the Heinemann maths scheme, they would be sitting with a book. I've got quite a few children in my class that completely zone out. Behaviour starts to get worse because they just don't enjoy doing it. And I don't have that problem anymore."
ASN Teacher with 8 years' experience

Teachers were asked in the final interviews to comment on the challenge of supporting particular children within the context of larger classes, and provided with the argument that it would be easier to remove those kids and do something different.

"…a lot of my day to day life would be easier if I just removed children out of the classroom. Like for any subject, not just for maths, but actually, it's not caused me any greater stress having them in. It's not really increased my workload having them in. Because when I sit and plan a problem with my planning sheet, I think, "well where do I want them all to be?", and have to consider maybe a few children either side of that, but the majority, I want to get to here, and I make slight allowances. "
Primary 2 teacher

Chapter 7: Conclusions and recommendations for practice

The project has successfully contributed to a statistically significant rise in numeracy attainment for primary school pupils across three schools in different local authorities. It has introduced 75 teachers to Cognitively Guided Instruction and supported them over a period of one to three years to introduce CGI to their classrooms and change the way they teach mathematics. The teachers worked with nearly 800 children. Assessment data from across all three schools showed a rise in attainment through school-based standardised assessment processes and crucially the pre- and post- assessments developed for the project showed a **growth in children's conceptual understanding**.

Throughout the project **we focussed on developing teachers' knowledge and understanding of children's mathematical thinking and applying that knowledge in practice**. **We did not focus explicitly on attainment -**this would seem to us to be a futile aim in the absence of any coherent activity which might lead to improvement in attainment. As the project progressed we were hearing increasingly from the teachers that the children in their classes were doing better, they had better understanding of the mathematics they were learning, they were engaging more fully with the maths and they were generally working beyond what was expected. These views were supported by teachers' judgements in their assessments of CfE levels and also in standardised assessments used in schools. In discussion with an education advisor for the GTCS (General Teaching Council for Scotland), this phenomenon was described by him as 'everything pointing in the same direction'. The final CGI assessments, which were administered at the very end of the project, provided robust statistical evidence through pre- and post-intervention comparisons of this rise in attainment and crucially demonstrated that this was connected to a growth in children's conceptual understanding. Qualitative data provided an explanation of the processes which contributed to this outcome. **The rise in attainment should be considered within the context of teachers' knowledge and the transformation of practice, as a consequence of understanding and responding to children's mathematical thinking.**

"With CGI pupils, learning is centred on the pupil's knowledge, whereas previously the pace was driven by the curriculum. By allowing more time for pupils to understand the concepts, we've then seen the pace of learning rocket. P3 are more advanced with fractions than they would be following the text book."
Primary 3 Teacher

There was evidence of an increase in teachers' knowledge about children's mathematical understanding and a consequent shift in the beliefs of many teachers. This led to a reconsideration of the role of the teacher and a pedagogical shift represented by a more child-led and guided approach and a move away from explicit direct instruction. **CGI challenged teachers' beliefs and offered teachers a means of testing regularly made assumptions. In response, teachers adopted key elements of CGI practice.**

Frequent Assumptions:	Key Elements of CGI Practice Adopted:
• Procedural competency is commensurate with conceptual understanding • Ability grouping supports children’s learning • Pupils will make the links between procedures and problem types • Counting activities and the use of materials belong in the lower primary • Pupils require explicit instruction of strategies and this is particularly the case for children with learning difficulties	• Belief that pupils come to school with intuitive knowledge of numeracy and can use that knowledge to make sense of problems and find their own solution strategies; • A more nuanced understanding of children’s mathematical thinking and how that helps teachers to set appropriate problems and guide children’s mathematical development; • Greater focus in early years teaching on understanding number and counting; • New approaches to supporting pupils to find and explain their own solution strategies; • Greater focus on dialogue and mathematical discussion, both in the classroom and with colleagues • More detailed and focussed observations of how particular children were attending to problems and using their analysis to inform instruction (Professional noticing) • More inclusive practice through a process of restructuring classroom practice • A pedagogical shift towards a guided approach

Teaching became informed by children’s thinking. Teachers were more aware of what to look for and they had a greater understanding of what they were seeing the children doing. They were beginning to use this to inform their teaching. **They were also highly creative and flexible in how they adapted this knowledge into their classrooms.** Initial concerns about ‘I don’t know if I’m doing it properly’ became displaced through communication and collaborative practice which helped teachers recognise their agency in developing CGI with integrity in their classrooms. **They recognised they needed to and were capable of taking ownership and that this was not a formulaic approach.** However, they also recognised this process as a learning journey that takes time and requires ongoing support.

The importance of recognising that effective and sustainable change takes time cannot be emphasised strongly enough. The project findings are consistent with previous studies that suggest that increasing teachers’ knowledge of children’s mathematical thinking can improve pupil attainment, but that **this is not a ‘quick fix’.** Teachers and schools need to make a considerable investment in terms of a commitment to learning, setting aside time for learning and co-ordinating professional learning in school and ensuring access to skilled and experienced practitioners. This needs to persist typically for 2-3 years to build knowledge and skills in individual teachers to become confident practitioners and for longer periods, 5-7 years, for those seeking to lead learning within their school.

Arguably, some current and recent development work in primary mathematics, including work involving CGI elsewhere in Scotland cited within the report, has been hampered by being led by practitioners with limited theoretical knowledge and experience in practice of particular programmes. The danger with such approaches is that they lead to the conclusion that a particular programme has been tried but found wanting when in fact the programme under scrutiny bears little resemblance to its original source. It was not within the scope of this project to evaluate the efficacy of the particular professional development models and programmes employed elsewhere, however, from the outset of the project we took a strong position on ensuring the integrity of CGI and that its development and any future development in Scotland should maintain fidelity to its underlying principles and philosophy.

Over a period of time we witnessed a transformation of practice. Fundamentally this was evident in a pedagogical and philosophical shift in the position of many teachers and a move away from what has been described as a transmission approach, towards a pedagogical orientation that supported children making connections in their mathematical understanding. **Teachers began to question the nature of children's mathematical knowledge and their own epistemological position in relation to children 'coming to know'. They reflected on the role of the teacher and how instruction might be designed deliberately and purposefully in order to support sense-making in tandem with computational fluency.** This kind of transformation is radically different to the acquisition of a range of techniques, strategies and ideas which come to be mapped onto existing practice.

In terms of the project's theory of change, we have learned a number of lessons that should assist development of a more refined model of teacher professional learning.

Inputs

Teachers of all ages and levels of teaching experience were, with very few exceptions, keen and interested to know more about children's mathematical thinking. This was seen as lacking in their initial teacher education programmes. For some teachers, CGI reinforced their own beliefs about children's intuitive abilities in mathematics, while others were more resistant until they saw it working in their colleagues' classrooms. Many teachers had identified gaps in children's conceptual understanding as opposed to procedural competency, but struggled to know how to respond until they learned more about CGI. Knowledge and skills in terms of professional noticing, designing word problems and leading mathematical discussion need to be developed in the classroom alongside hands on experts.

In the course of the project we dynamically assessed changes in teachers' knowledge and beliefs at each school using the classification framework described in Chapter 4. By the end of the project we found that most teachers were operating at Level three. This was evidenced in the interviews and professional dialogue by an expressed belief that pupils can solve problems without explicit instruction. Classroom practice was dominated by pupils solving problems and discussing solution strategies. Over time teachers were designing more appropriate problems and asking and focussed questions to understand children's thinking. For many teachers this represented a significant turning point and was where we would hope them to be by the end of the project.

Professional development interventions

Teachers learned from both accredited learning and in-service seminars, but **there was no evidence that one type of learning opportunity was more effective than the other in terms of preparation for introducing CGI to the classroom and neither was sufficient**. In terms of situated learning in the classroom, teachers benefited from seeing CGI practice directly, however, **collaborative development of lesson study and delivery was far more effective than classroom demonstrations by experts**. Professional dialogue with experts and with each other was a vital part of creating a professional learning community that could continue to support and develop CGI practice.

Classroom Practice

Classroom practice had to change. Teachers needed to become more aware of the individual development of children's mathematical thinking through dialogue and dynamic assessment. Lesson planning had to become more child-centred and mathematically focused. The Lesson Study planning framework and observation schedule assisted significantly with this process.

Early years teachers found it easier to embrace CGI than did upper primary school teachers, perhaps in part because teaching became more inclusive. In contrast to the previous practice of teaching a specific algorithm which pupils may or may not intuitively understand, by encouraging pupils to make sense of mathematics for themselves and inviting a wide range of solution strategies, pupils were less likely to 'fail'.

Lesson Study

As the project progressed Lesson Study became the dominant model for supporting teachers' learning. **It provided a collaborative and reflective framework to which teachers were highly responsive**. Teachers commented on the importance of a having a knowledgeable and experienced practitioner of CGI involved in the Lesson Study, without whom there was a danger that key points would be over-looked or misinterpreted. All three schools embraced this model. It was notable that teachers began to organise their own lesson study cycles independently of the project, continuing with it after fieldwork with the schools had ended. **Headteachers and members of the school leadership teams also stated that they intended to use this model for the development of other areas of the curriculum.**

School-based co-ordinators

The co-ordinators had a practical function in terms of supporting the day-to-day management of the project in the schools and they excelled in this role. However, there was a fundamentally more significant purpose to their role in relation to ensuring the long-term sustainability of the work initiated in the schools. **It was intended that future leaders of professional development in CGI should emerge from within the schools**. **These leaders need to be nurtured and supported**. The co-ordinators recognised this and were highly reflective in terms of their own learning. They collaborated as a team across schools in a very effective manner.

Impact on pupil attainment

There was a statistically significant rise in some areas of pupil attainment and an improvement in almost all other areas across all three schools evidenced in the pre- and post- intervention assessments. This was mirrored in school-based assessments and teacher judgements. Pupils became more engaged in mathematics, showing increased confidence, independence and flexible thinking. They were able to apply what they already knew to help them tackle problems of increasing complexity. Pupils in upper primary classes rely most on teachers supporting them to make links between their own strategies and what are considered the standard algorithms seen as required for secondary education.

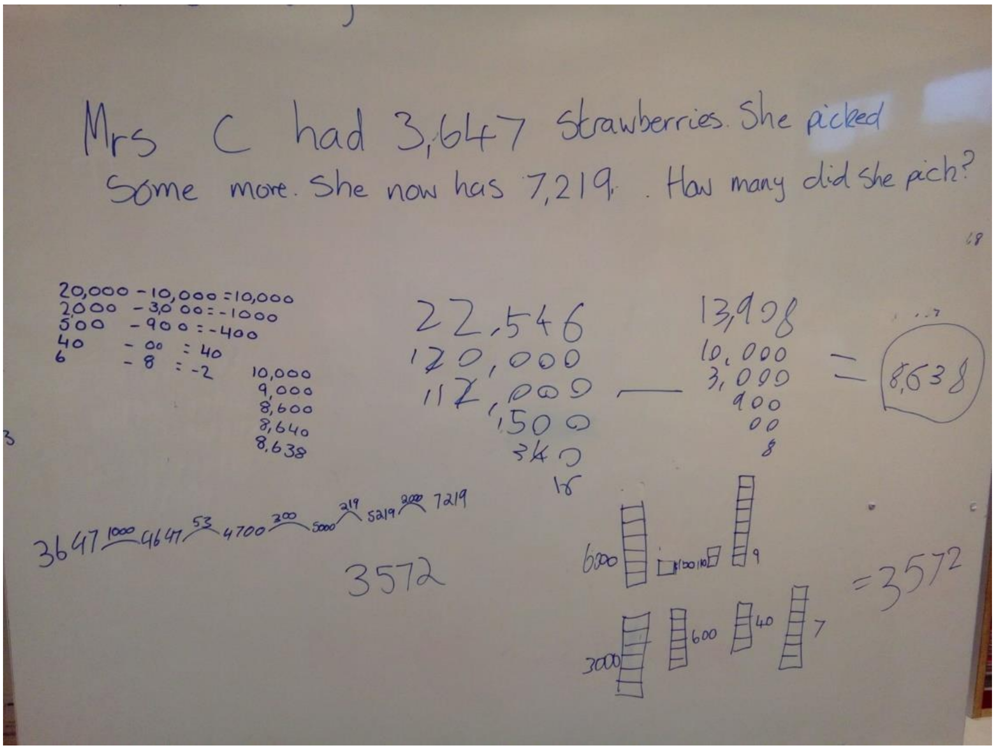


Figure 17 Primary 7 linking solution strategies and standard algorithms

Future Models of Teacher Professional Learning

The project was informed by the conceptual framework set out in Chapter 3. Supporting teachers' learning was at the heart of our work and in this respect we worked to a set of organising principles which have been borne out by the project,

Organising principles

- Needs to be situated, meaningful, supported
- Needs to focus on children's thinking
- Needs to be inclusive
- Needs to focus on pedagogy
- Needs to be about knowledge
- Needs to be respectful of teachers' prior knowledge and ways of learning
- Needs to be mathematically informed
- Needs to be part of a professional learning community
- Needs to be long term

Why this matters

Schools are currently under considerable pressure to raise attainment in numeracy. Our study suggests that a rise in attainment can come about by focussing on teachers' knowledge and pedagogy and specifically, teachers' pedagogical content knowledge, rather than by focussing on attainment itself. There is a moral imperative to this argument - children's mathematical learning needs to equip them for life, not assessments. Focussing on children's conceptual understanding does not need to be at the expense of computational fluency but teaching children in ways that ensure that they understand the maths they are learning can accomplish both. Many adults will struggle to recall how to calculate 2½ ÷ ¼, few will struggle to work out how many people can get quarter of a pizza if there are two and half pizzas. Teaching children in ways that allow them to connect their intuitive mathematical understanding to the formal mathematics of school is the task of the accomplished teacher. We need to support teachers to develop their capacity in this role.

Recommendations

Teacher learning needs to be embedded in classroom practice. Having laid the foundations for practice in each of the schools they are now in the position to work collaboratively within their schools to extend that practice. We offer the following recommendations for school-based development:

1. Focus on depth of understanding and quality of practice
2. Ensure learning is underpinned by knowledgeable and experienced proponents committed to principles of CGI and the organising principles above
3. Encourage as a pedagogy for **all** children and reflective of inclusive and equitable classrooms

4. Give teachers time and opportunity to develop that practice in their own classroom **and then listen to them**
5. Support the development of key practitioners (co-ordinators) within the school as eventual leaders
6. Support teacher learning embedded in the classroom and use classrooms as contexts for situated and collaborative learning
7. Use a Lesson study model to support this
8. Ensure knowledgeable and experienced proponents of CGI are involved in the Lesson Study cycles
9. Work with whole schools and ensure the buy-in and active engagement of school leadership teams
10. Don't assume that this can happen quickly

Scaling up - Where do we go from here?

> *"From the ground up makes good sense for building. Beware of from the top down."* *Frank Lloyd Wright*

We described our strategy at the outset of the project as 'an inch wide and a mile deep'. We deliberately worked with a small group of schools and took a long-term view of scaling up this work. Nonetheless, the objective is that over time schools become satellite CGI learning communities capable of not only inducting new teachers to the practice, but of continually learning and developing CGI thinking by connecting with others beyond their own school.

The initial investment in concentrated in-service support, gradually gives way as centres of strong practice emerge and these can be used as the basis for further development within their learning communities in an organic way. Professional learning communities potentially provide a useful model for such development. The ten recommendations listed above are applicable to extending practice to the wider community, however we would stress the following:

- Ensure professional development is led by knowledgeable and experienced practitioners
- Support the development of school-based leaders
- Work gradually and with due attention to the integrity of CGI principles
- Recognise that this takes years

The development of this work should also been seen within the context of the wider CGI learning community in Scotland. Over the last few years many teachers have had an introduction to CGI through professional development. There is a growing body of teachers eager to learn about CGI and share and extend their practice, exemplified by the activity on the Cognitively Guided Instruction in Scotland Facebook page. Consideration should be given to how we support and grow this community.

Future research

In light of statements from teachers in our study in relation to teachers' mathematical knowledge and preparation for teaching, it would be useful to investigate not only the amount of time given to primary mathematics in Initial Teaching Education courses, but more specifically the content and usefulness of that content for practice.

Further research to verify the relationship between the introduction of CGI and improvements in pupil attainment would benefit from comparison with control groups and tracking teachers and pupils over a longer term to see whether early gains are sustained.

The embedded nature of this work also provides rich opportunities for further research, not addressed in our study, which could usefully focus on:

- teacher self-confidence and efficacy;
- management dynamics and leadership of change
- comparison of CGI assessments and SNSAs
- Professional learning communities

Limitations

Although the study involved a significant number of children and teachers, it was in the context of three schools and we are reporting on the basis of our experience with these three schools. Schools are dynamic organisations each with their own particular challenges, visions and ethos, we recognise this and are cautious of any over-generalisation of our findings.

At the outset of the project each of the schools had its own assessment tool for numeracy making cross-school comparisons problematic. The CGI assessment tool which we piloted across the schools was effective in assessing children's understanding pre- and post-intervention but it does not permit a comparison with the wider population. Our study was coming to a close at a period when standardised national assessments (SNSAs) were being introduced. Future studies may be in a position to make more comparative use of SNSA data.

Given the limited pool of CGI practitioners in the UK at present, further development would need to be planned strategically over the longer term. On the basis of lessons learned in the course of this project, were we to carry out similar projects in the future we would move to a Lesson Study model sooner.

The last word

The project came about at the initiative of the various headteachers who were eager to develop practice in the teaching of numeracy in their schools and who were able to make use of Scottish Attainment Challenge and Pupil Equity Fund to support the development work. They summarised their evaluation of the project,

> *"It is not just about the children's attainment, it is about upskilling the staff and those for me have been the two biggest impacts that I have seen in my school."*
> *Maria Seery, Headteacher, 2018*

“The teachers’ knowledge of mathematical thinking has developed so greatly and that has had a great impact on our children.”
Martine Watt, Headteacher, 2018

“It’s a more holistic approach to assessing and it’s an ongoing process rather than a one-day snapshot of their achievements. What the data is telling us is that there has been a massive increase in the attainment of numeracy across the whole school.”
Kate Fisher, Headteacher, 2018

Post-script

Schools have a natural turnover of staff, this can vary between periods of stability and consistency to sudden and rapid changes to school personnel. Since the end of the project in June 2018, we have learned that there has been a significant change of staff in some of the schools. A considerable number of staff who had undergone professional development in CGI in at least two of the schools have moved on for various reasons. This presents an obvious challenge to ensuring the sustainability of the work initiated in those schools. Without ongoing support and a strategic approach towards the induction of new members of staff there is a danger that the strong foundations which have been laid will be gradually weakened and ultimately the work will washout. Our intention would be to continue to build capacity within the schools through our work with the co-ordinators so that they might reach a point where they would be self-maintaining.

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